Granger Causality of the Inflation Growth Mirror, the Case for Mexico

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Abstract

Analytical and econometric models are developed where exogenous money supply causes changes in inflation and output growth rate. It is found that money through the velocity of money is important for determining output growth and inflation structural breaks. The analytical model explain these facts by considering inflation as a tax on the return to human capital, which in turn induces a growth rate decrease. The empirical model uses Mexican data and panel Vector Autoregressive methodology to expound the equilibrium path dynamics for leading macroeconomic indicators: money growth, inflation and output growth. Granger causality tests support the finding that money Granger causes inflation and inflation Granger causes output.

Introduction

This document follows closely Gillman and Nakov (2004) analytical and empirical model to study the relationships among the leading macroeconomic indicators on Hungary, Poland, Romania and Bulgaria. Gillman and Nakov (2004) model investigates the equilibrium path of these economies, taking into account the income velocity of money as endogenously determined by the relative cost of money *vis-à-vis* the credit cost which is produced in a separate 'banking' sector.

Descriptive statistics are presented in section 2, which is devoted to describe the data. The purpose of this section is to provide a snapshot on the evolution of the leading macroeconomic indicators under study: money growth, inflation and output growth. These descriptive statistics are composed by the summary statistics and figure analysis. In the figure analysis sub-section, it is found that for the Mexican case a phenomenon called the 'transition mirror' holds. Gillman and Nakov (2004) study the 'transition mirror' for the case of Hungary, Poland, Romania and Bulgaria. These authors explain that given this phenomenon, inflation almost mirrors output growth. In other words, there is a negative relationship between inflation and output that evolves almost on inverse proportion.

The empirical results section reports the panel VAR short-run dynamics results and its impulse response functions.[3] It is found that the transmission mechanism from the analytical model to the real economy is set through the velocity of money evolution and its impact on leading macroeconomic indicators, as the analytical model of Gillman and Nakov (2004) have already predicted.

1. The Analytical Model

This study follows closely the model develop by Gillman and Nakov (2004) and its extensions on Gillman and Kejak (2005). The aim of this document is to apply this analytical model and test its empirical implications for the Mexican case. Gillman and Nakov (2004) develop an analytical monetary endogenous growth model. In this model human capital accumulation leads to economic growth.[4] Also, it introduces a modified cash-in-advance transactions technology.[5] An important feature of this model consists on credit extension usages *i.e.*, consumption and investment, with symmetric weights.

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[2] I would like to acknowledge Professor Max Gillman for all his guidance and comments towards this research and Central European University for providing me with exceptional facilities to conclude this document.

[3] The Appendices contain the Augmented Dickey-Fuller and Breakpoint Unit Root tests; data sources and transformations. Also, Granger causality tests results are reported in this section.

[4] Lucas (1988).

[5] Lucas (1980).

This closed-form solution can account for structural breaks in leading macroeconomic variables. Moreover, in this solution the velocity of money and structural breaks movements are linked with the banking sector productivity.
The analytical model sets the general equilibrium paths for all sectors in the economy, including the banking sector. The banking sector money supply growth is set on terms of output growth and inflation targets.[6] The equation for money demand provides the analytical mechanism in how money; output growth and inflation are related with the banking sector productivity and the velocity of money. The transmission mechanism uses the velocity of money to determine inflation and output growth. Therefore, the bridge between the analytical model and empirical part is the velocity of money. That is to say, by testing econometrically the relationships among the variables under study, it is found that they are related as the analytical model have predicted.

1.1 Consumer problem

The consumer utility function is expressed in a linear fashion, as:

$$u_t = lnc_t + \alpha lnx_t \qquad (1)$$

Where u_t stands for utility; c_t stands for consumption; x_t stands for leisure; ln stands for logarithm; α stands for the participation that leisure have in utility and t stands for time.[7]

The time allocation constraint is:

$$1 = l_t + l_{ht} + l_{dt} + x_t \qquad (2)$$

where 1 stands for a normalize day (24 hours divided by 24 hours); l_t stands for time spent working in the goods production sector; l_{ht} stands for time spent to produce human capital and l_{dt} stands for time spent to produce credit or 'banking time.'

The income constraint is:

$$r_t k_t + w_t l_t h_t + v_t - c_t - \dot{k}_t - \delta_k k_t - \dot{m}_t - \pi_t m_t = 0 \qquad (3)$$

where $r_t k_t$ stands for real rental income; $w_t l_t h_t$ stands for real wage income from effective labor; h_t stands for human capital stock; $\dot{k}_t + \delta_k k_t$ stands for physical capital investment; $\dot{m}_t$ stands for real money stock investment and π_t stands for inflation rate.

Real lump-sum transfers from the government to the consumer adds to the income constraint. This subsidy is equal to the inflation tax proceeds, which is defined as:

$$v_t = \frac{V_t}{P_t}$$

where V_t stands for velocity of money and P_t stands for prices. The human capital production function is:

$$\dot{h}_t = (1 - \delta_h) h_t + A_h l_{ht} h_t \qquad (4)$$

where $\dot{h}_t$ stands for the logarithm growth rate of human capital stock. The equilibrium is reached when:[8]

$$y_t = c_t + \dot{k}_t + \delta_k k_t$$

where y_t stands for the production function or total income; $\dot{k}_t$ stands for the logarithm growth rate of the physical capital and δ_k stands for the rate of physical capital depreciation.

The output on monetary terms can be expressed as follows:

$$m_t + d_t = y_t \qquad (5)$$

where d_t stands for real credit purchases and m_t stands for real money purchases.

The credit technology is described as follows:

$$d_t = A_{dt} (l_{dt} h_t)^{\gamma} {y_t}^{1-\gamma} \qquad (6)$$

[6] In general equilibrium money demand equals money supply. For instance, in the banking sector optimum it is seamless to talk about money supply or money demand.

[7] Henceforth, the sub-index τ stands for time.

[8] Either using money or credit.

Where γ takes values on the interval (0, 0.5).[9] The share of purchases made with money is defined as identity as follows:

$$a_t \equiv \frac{m_t}{y_t}$$

Taking into account the above identity the 'Clower constraint' for monetary economies is:

$$m_t = a_t y_t \qquad (7)$$

The credit share can be found by substituting equations (6) and (7) in (5):

$$(1 - a_t) = A_{dt}\left(\frac{l_{dt}h_t}{y_t}\right)^{\gamma} \qquad (8)$$

Solving the system of (7) and (8) for m_t gives back the revised 'Clower constraint':[10]

$$m_t = \left[1 - A_{dt}\left(\frac{l_{dt}h_t}{y_t}\right)^{\gamma}\right] y_t \qquad (9)$$

By substituting equation (6) in equation (9) yields:

$$m_t = y_t - d_t$$

Equation (9) includes the banking time in terms of credit technology. Money demand and its velocity can be obtained from equation (9).[11]

1.2 Goods producer problem

The goods production function is assumed to have constant returns to scale with respect to capital and labor. The constant returns to scale technology of the production function are taking into account by using a Cobb-Douglas functional form, as follows:

$$y_t = A_g (l_t h_t)^{\beta}(k_t)^{1-\beta} \qquad (10)$$

Where A_g stands for a technology parameter and β is the output share for human capital. It takes values on the open interval (0, 1) and $(1 - \beta)$ is the output share for physical capital.

The first order conditions are:

$$w_t = \beta A_g (l_t h_t)^{\beta - 1}(k_t)^{1-\beta} \qquad (11)$$

$$r_t = (1 - \beta) A_g (l_t h_t)^{\beta}(k_t)^{-\beta} \qquad (12)$$

1.3 Government money supply

The government provides money supply through lump-sum tax transfers. Its functional form is:

$$\dot{M}_t = V_t \qquad (13)$$

It is assumed that equation (13) has a constant rate of growth equal to σ.

1.4 Equilibrium

The Hamiltonian for the above consumer and producer system is:[12]

$$H = e^{-\rho t}(lnc_t + \alpha lnx_t) + \lambda_t\left(r_t k_t + w_t l_t h_t + v_t - c_t - \dot{k}_t - \delta_k k_t - \dot{m}_t - \pi_t m_t\right)$$
$$+\eta_t\left[A_{lt}(1 - l_t - l_{dt} - x_t)h_t - \delta_h h_t - \dot{h}_t\right]$$
$$+\mu_t\left\{m_t - \left[1 - A_{dt}\left(\frac{l_{dt}h_t}{A_g(l_t h_t)^{\beta}(k_t)^{1-\beta}}\right)^{\gamma} A_g(l_t h_t)^{\beta}(k_t)^{1-\beta}\right]\right\} \qquad (14)$$

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[9] This implies a convex marginal costs that rises as output increases.
[10] The Clower constraint is also known as the exchange constraint.
[11] This result can be achieved by following Baumol (1952) procedure.
[12] The Hamiltonian helps in determining the dynamic solution for a set of differential equations. The Halmitonian discrete version is the Bellman equation.

Maximizing with respect to c_t, x_t, l_t, l_{dt}, m_t, k_t, h_t renders the Hamiltonian first order conditions. Rearranging these conditions in reduced-forms provides the general equilibrium expressions, for goods and leisure; return of human and physical capital (balanced growth path g); nominal interest rate (implicit Fisherian equation R_t); a closed-form solution for a_t and money demand, as follows:

$$\frac{x_t}{\alpha c_t} = \frac{1+R_t[\gamma+a_t(1-\gamma)]}{w_t h_t} \tag{15}$$

$$-\frac{\dot{\mu}_t}{\mu_t} = A_h(1-x_t) - \delta_h = r_t\left\{1 - \frac{a_t R_t}{1+R_t[\gamma+a_t(1-\gamma)]}\right\} - \delta_k = -\frac{\dot{\lambda}_t}{\lambda_t} \tag{16}$$

$$g = A_h(1-x_t) - \delta_h - \rho = r_t\left\{1 - \frac{a_t R_t}{1+R_t[\gamma+a_t(1-\gamma)]}\right\} - \delta_k - \rho \tag{17}$$

$$R_t = r_t - \frac{a_t R_t}{1+R_t[\gamma+a_t(1-\gamma)]} - \delta_k + \pi_t \tag{18}$$

$$m_t = \left[1 - A_{dt}^{\frac{1}{1-\gamma}}\left(\frac{\gamma R_t}{w_t}\right)^{\frac{\gamma}{1-\gamma}}\right] y_t \tag{19}$$

$$a_t = 1 - A_d^{\frac{1}{1-\gamma}}\left(\frac{\gamma R_t}{w_t}\right)^{\frac{\gamma}{1-\gamma}} \tag{20}$$

When $a_t = 1$ equation (15) describes a money-only economy. In this case, the shadow price of goods is one, plus the nominal interest rate R_t, for all type of purchases. With the presence of credit the exchange cost is $a_t R_t + (1-a_t)\gamma R_t$. In this expression, the average cost of money is R_t and for credit is γR_t. The corresponding weights are a_t and $(1-a_t)$.[13]

In equation (17), endogenous factors could affect the income velocity of money.[14] For instance, with less money more credit is used. This shift increases the goods costs relative to leisure costs. Thus, the agent substitutes goods for leisure. This causes the return to human capital and growth to drop. All these steps trigger the transmission mechanism embedded in equation (19) *i.e.*, inflation lowers the output growth rate by means of a lower return to human capital rate. Inflation and other endogenous factors could increase the income velocity of money $\frac{y_t}{m_t}$ on equation (19). These endogenous factors may also explain structural breaks by shifting the parameter A_{dt}.

The transmission mechanism embedded in equations (17) and (19), their equilibrium paths and optima are going to be tested by means of a panel Vector Autoregressive (VAR) model. The econometric results are reported in section 3. Granger causality tests reported in Appendix 3.2 support the VAR empirical findings.

2. The data

2.1. Descriptive statistics

By means of descriptive statistics, the principal trends of the macroeconomic indicators under study are examined. For practical purposes, the almost 30 years of data are segmented in four not overlapping time sub-periods *i.e.*, 1986:Q1-1993:Q4; 1994:Q1-2000:Q4; 2001:Q1-2007:Q4 and 2008:Q1-2016:Q1.[15] These sub-periods are almost symmetrical with respect to their number of observations. That is to say, they are 32; 28; 28 and 33, respectively.

First, Table 1 presents summary statistics for output; inflation and money. These statistics are composed for the principal central moments for each macroeconomic indicator. The statistics reported are the first (*Mean*); second (Standard Deviation, SD) and fourth (Kurtosis, KT) moments for the variables under study.
The Coefficient of Variation (CV) is put forward in this table.[16] Second, a figure analysis is implemented for the whole sample, which takes almost 30 years of data, as well as for four not overlapping sub-periods. The end of this section presents a brief summary.

[13] These weights allow expressing income velocity of money in a closed-form solution.

[14] These factors are endogenous to the system, but exogenous for the variables they are determining.

[15] The date for the first time sub-period is 1986:Q1-1993:Q4. This date should be read: from the first quarter of 1986 to the fourth quarter of 1993.

[16] The second moment is the variance (Table 1 above reports the variance square root, or standard deviation). CV is the ratio between standard deviation and mean, CV=SD/*Mean*.

Table 1: Summary statistics. Leading macroeconomic indicators. Mexico. Selected periods

		1986:Q1-1993:Q4	1994:Q1-2000:Q4	2001:Q1-2007:Q4	2008:Q1-2016:Q1
Output					
	Mean	62.93	78.34	93.09	102.04
	SD	6.03	8.08	5.23	4.85
	KT	1.62	1.75	1.81	2.52
	CV	0.10	0.10	0.06	0.05
Inflation					
	Mean	10.17	41.17	75.39	103.78
	SD	5.60	14.93	6.47	9.50
	KT	1.78	1.73	1.81	1.83
	CV	0.55	0.36	0.09	0.09
Money					
	Mean	49*	278*	851*	2,040*
	SD	47*	124*	215*	605*
	KT	2.03	1.85	1.89	2.20
	CV	0.97	0.45	0.25	0.30
	n	32	28	28	33

Note 1: Output stands for the manufacturing production index 2008=100; Inflation stands for the national consumer price index 2010=100; Money stands for M1 currency held by the public and short-run bank deposits. All variables are in levels;
Note 2: Q1 stands for first quarter; Q4 stands for four quarter; * stands for millions of Mexican pesos and n is the number of observations.

Source: Own estimates based on INEGI (National Institute of Statistics and Geography), using E-Views 9.0.

In Table 1 it can be seen that *Output* has been increased across time *i.e.*, 62.93; 78.34; 93.09 and 102.04. Its standard deviation remains relatively flat with a downward trend at the end of the whole period. An exception for this trend is presented for the second period *i.e.*, 8.08. The kurtosis describes increasing output persistence:[17] 1.62; 1.75; 1.81 and 2.52 for each sub-period, respectively. This trend implies that output process is becoming stationary through time. This output stability is also seen in its CV figures, since they take on values of 0.10 for the first two sub-periods and decrease to 0.06 and 0.05 in the last two sub-periods.

For its part, the *Inflation* means have values of 10.17; 41.17; 75.39 and 103.78 for each sub-period, respectively. Its standard deviation has also experienced increases during these four sub-periods, where 5.60 are the lower value for 1986-1993 period and 14.93 is the highest value across time. The standard deviations are 6.47 and 9.50 for the last two sub-periods.[18] Its kurtosis statistic remain relatively flat during all four sub-periods, but with a slightly increase towards the last sup-period *i.e.*, 1.78; 1.73; 1.81 and 1.83. For their part, Chiquiar and Noriega (2007) documented a disinflation process as a global phenomenon, given an increase in persistence. These authors also note that the use of nominal inflation anchors increase inflation stationarity.[19] Similarly, the Coefficient of Variation experiments drastic decreases across time. It registers a value of 0.55 during the 80's, moving to 0.09 in the last two sub-periods. During 1994:Q1-2000:Q4, the CV value (0.36) have a transition period, between high volatility in prices (0.55), in the first sub-period and a relatively stable price evolution (0.09) towards the last sub-period.

Money for its part has been nominally growing from a modest mean of 49 millions of Mexican pesos, for the sub-period of 1986:Q1-1993:Q4 to 2,040 million for the sub-period of 2008:Q1-2016:Q1.

[17] The higher the kurtosis statistics, the higher the persistence in its process. A higher persistence indicates fatter tails or an increasing stationarity in the macroeconomic indicator distribution. According with R. Douc, *et. al* (2014) kurtosis is associated with a strict stationary solution. For Arellano and Bond (1991), kurtosis, the fourth-order moment of the data indicates the convergence velocity to normality.
[18] An increasing trend is observed in these two last sub-periods, with respect to the first period value.
[19] For instance nominal rigidities *i.e.*, nominal exchange rate crawling peg. These authors address the reduction of persistence on higher levels of inflation, by noting increases on stationarity.

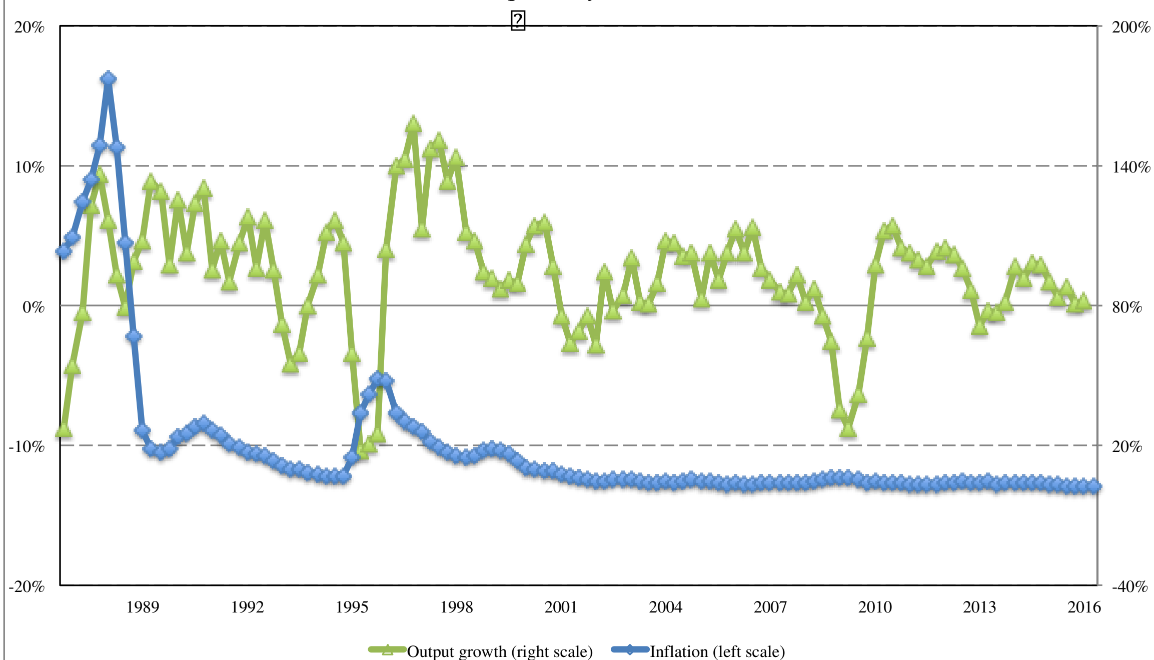
Its standard deviation has increased more than 1,000% through time, as it passes from 47 millions on the first sub-period to 605 millions in the last sub-period.[20] The kurtosis statistic has increased from 2.03 to 2.20 across time, with a slightly reversion in its trend during the 90's and beginning of 00's *i.e.*, 1.85 and 1.90, respectively. *Money* CV has decreased almost two thirds *i.e.*, 0.97 to 0.30 for the first and fourth time sub-periods. This CV decrease is observed given a higher mean growth rate, with respect to the one for standard deviation.

2.2. Figure analysis

There is a wide heterogeneity in the time availability for output growth, inflation and money growth for the Mexican case.[21] However, there is a homogenous and continuous quarterly time period for these three leading macroeconomic indicators, which goes from the fourth quarter of 1986 to the second quarter of 2016. Next, Figure 1 shows the year growth rate on a quarterly basis, for prices growth (inflation) and output growth.[22]

Figure 1. Mexico: Inflation, output growth. One year percentages

1987-2016 (quarterly data)



Source: Own computations based on INEGI (National Institute of Statistics and Geography).

In Figure 1 the 'transition mirror' phenomenon, described in Gillman and Nakov (2004), is found for the Mexican case. The time sub-periods for the 'transition mirror' are 1989 to 1995; 1995-1996 and 1996 to 2008. As inflation remains relatively flat in Figure 1 for the last years *i.e.*, after 2008, it is hard to discern whether output and inflation have a mirror relationship. For practical purposes and for increasing legibility, Figure 1 is divided in four not overlapping sub-periods.[23] In what follows, it is presented Figure 1a for the time period of 1987:Q1-1993:Q4.[24] Similarly, Figure 1b is for 1994:Q1-2000:Q4; Figure 1c is for 2001:Q1-2007:Q4 and Figure 1d is for 2008:Q1-2016:Q1. The sub-periods, besides of being practical, basically match four important landmarks in the Mexican economy evolution.[25] These landmarks are the privatization of the Mexican banking sector on 1989; the1994 'tequila crises' associated with a drastic Mexican peso devaluation; 2001 U.S. middle recession and the 2008 'great financial crises.'[26]

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[20] The corresponding percentage increase is: (605/47)*100=1,287%.

[21] See Appendix 1 for a report on the macroeconomic indicators time availability.

[22] See Appendix 2 for details in data transformations.

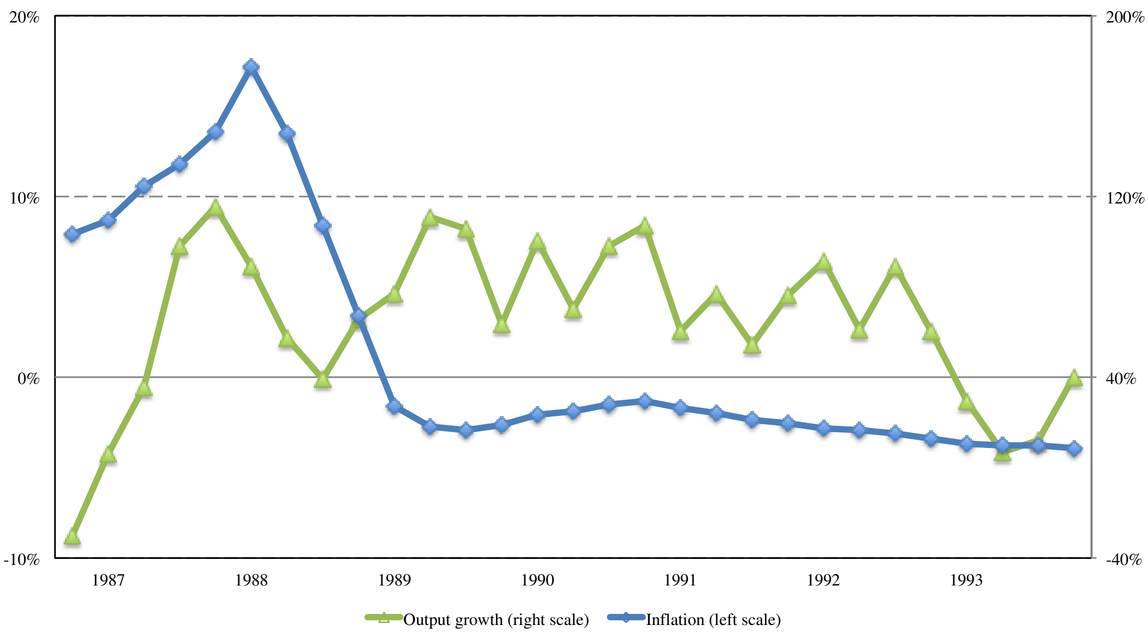
[23] These sub-periods match those reported on Table 1.

[24] The figures use one-year percentage growth. Thus, the first four quarters are lost.

[25] An attempt is made to develop a narrative approach to explain shifts or structural breaks in the time series under study.

[26] As the analysis develops, it could be found that some institutional factors shifts might match shifts in output growth and inflation, during the selected time sub-periods.

Figure 1a. Mexico: Inflation, output growth. One year percentages
1987-1993 (quarterly data)



Source: Own computations based on INEGI (National Institute of Statistics and Geography).

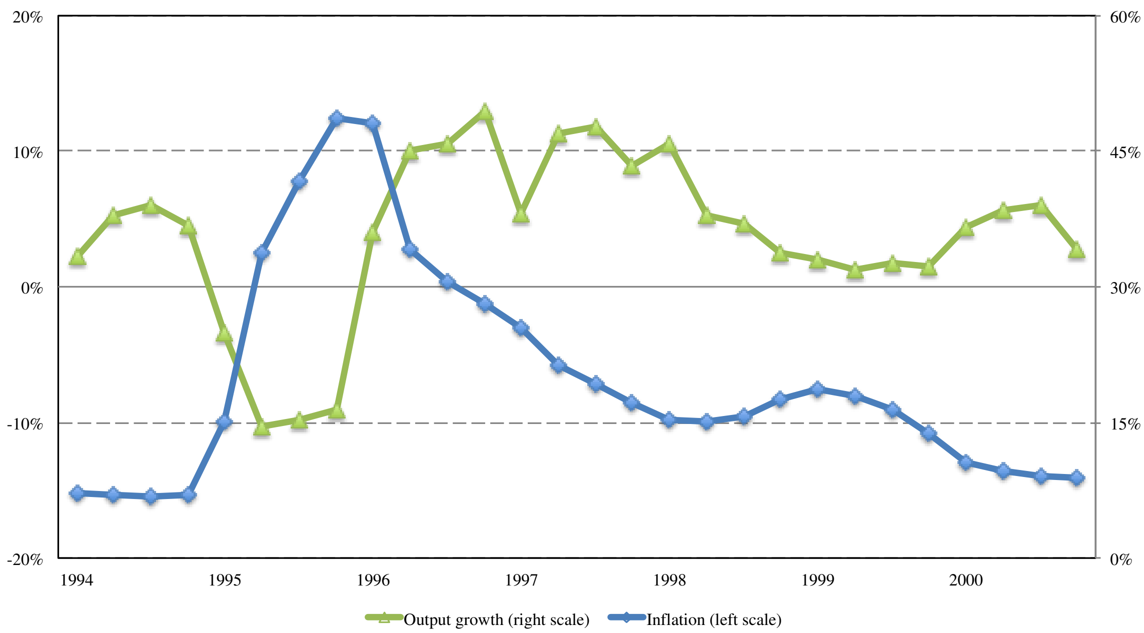
In Figure 1a, the years of 1987 and 1988 inflation and output growth do not seem to have an inverse relationship. When inflation and output cross in 1989 the "transition mirror" phenomenon appears and stays during all time period of 1989-to the end of 1993.

Moreno (2007) reports that Mexico changed its developing strategy in mid-1980's from an import substitution and state-led industrialization, in favor of a strategy centered on trade and financial liberalization, plus a drastic reduction of the state's intervention in the economy. As part of the state size reduction, the privatization of national banks took place. Bank privatization seems to be a good candidate for explaining why output growth and inflation cross during 1989.

The institutional adjustments described above had the objective to set the Mexican economy on a non-inflationary, export-led growth path driven by sales of manufactured goods. However, Figure 1b despites a drastic drop on output growth, given the Mexican peso devaluation. Perhaps, this event is signaling that output growth was driven for an exchange rate nominal anchor.[27]

[27] By keeping an exchange rate nominal anchor, imports are made artificially cheaper. Mexican manufacturing production relies heavily in imported intermediate materials. The whole effect is that Mexican exports become internationally price competitive, given a nominal anchor. For more information in this regard see Blecker (1996).

Figure 1b. Mexico: Inflation, output growth. One year percentages
1994-2000 (quarterly data)



Source: Own computations based on INEGI (National Institute of Statistics and Geography).

Figure 1b depicts the 'transition mirror' phenomenon for all the time sub-periods under consideration. The time series of inflation and output growth cross in 1995 and at the end of 1996. During these two years, output experimented sudden changes: from negative numbers *i.e.*, -10% as part of the 'tequila crises' towards positive numbers of a magnitude of 10%. Thus, output growth rate variation during 1995-1996 was about 200%. Inflation reaches at the end of 1996 its higher growth rate (45%). Afterwards, on 1996 inflation experimented important growth rate decreases, which are near to 10% at the end of 2000.

The 'tequila crises' started at the end of December, 1994. The Mexican peso *vis-à-vis* the U.S. dollar experienced a sudden devaluation. This devaluation caused inflation to spike during 1996.[28] The effects of large devaluations over inflation are transmitted by means of importable tradable goods prices.[29] A second round of structural adjustment was implemented during the 90's. The decrease of fiscal debt and inflation stability, that accompanies this second round of privatizations, were a condition for the U.S. government to lend the Mexican government nearly 30 billions of dollars to palliate devaluation effects. Although, this loan was paid back in its entirety in less than one year, it served to backup the confidence on the Mexican peso.[30]

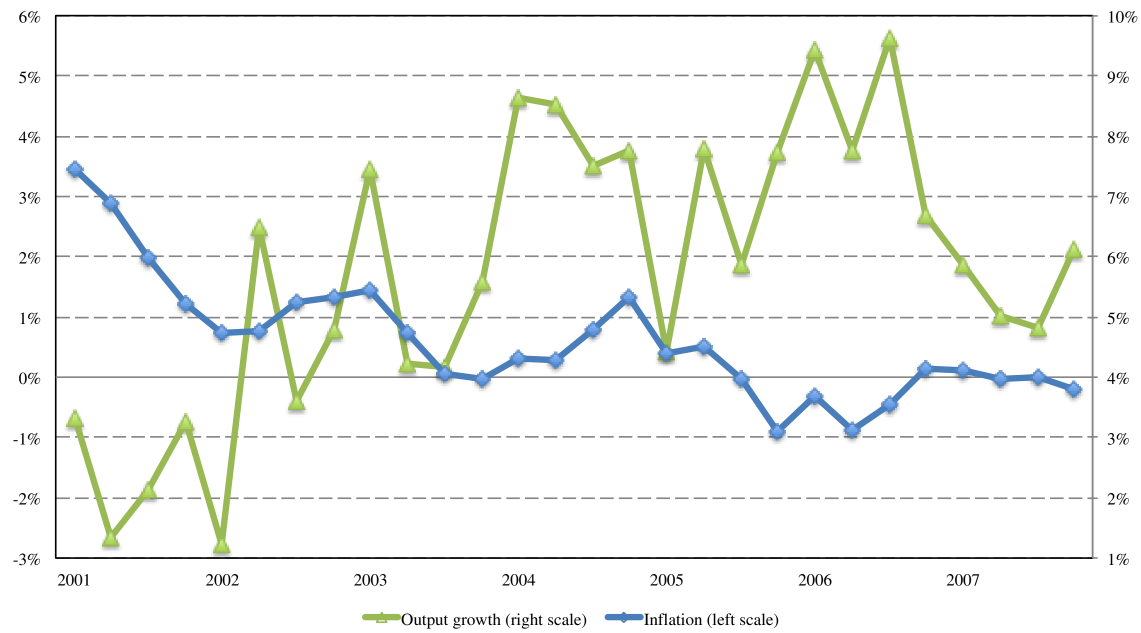
[28] According to Blanchard and Fisher (1989) appreciation under rational expectations implies that the monetary authority uses the real nominal exchange rate as an anchor for domestic prices, while at the same time, it makes imports cheaper in relative terms. If the nominal anchor disappears, inverse effects to those explained at the beginning of this footnote are expected.

[29] For a comprehensive treatment on devaluations and their impact in price indexes, see Burstein *et. al* (2005).

[30] For a review of these facts sequence, see Gil-Diaz and Carstens (1996). The first author was the Mexican Central Bank governor at the time of the 'tequila crises.' The second author is currently the governor of the Mexican Central Bank.

Figure 1c. Mexico: Inflation, output growth. One year percentages

2001-2007 (quarterly data)



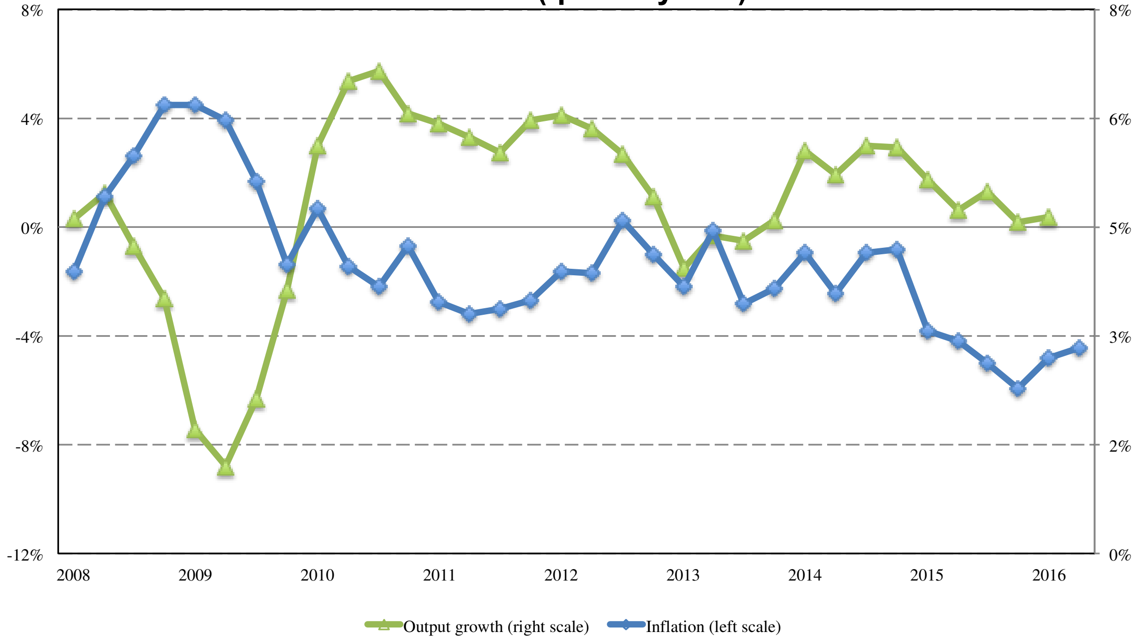
Source: Own computations based on INEGI (National Institute of Statistics and Geography).

Figure 1c shows the time sub-period between 2001 and 2007. During these years the time series of inflation and output crosses five times: at the middle and end of 2002; at the beginning, middle and end of 2003. At the beginning of 2005 inflation and output growth do not cross, but touch. During the time period of 2003-2007, the 'transition mirror' is clearly seen in Figure 1c. Perhaps, the strong economic linkages between U.S. and Mexico could explain some of these crosses. For example, in 2001 there was a middle recession in the U.S. economy. Some economic analysis linked this recession with China entering the World Trade Organization (WTO), using quotas and privileges to compete in the American market. Thus U.S. manufacturing output dropped in important industries, *i.e*. textile.[31] Perhaps, the lack of markets caused the U.S. manufacturing output to decrease. This drop on output U.S. seems to be transmitted, with some lags onto Mexican growth output *i.e.*, 2002. During 2006 and part of 2007 the U.S. economy experienced the first effects of the 'great financial crises.' These effects may be reflected in the Mexican output growth rate decrease during the middle of 2006 towards the end of 2007.

According to Capistran and Lopez-Moctezuma (2010) there was 'good news' around 2002 to 2007, which meant positive shocks for the Mexican economy. These authors argue, that the Mexican Central Bank might have got stronger by this news and reinforce its efforts to transit to an inflation-targeting regimen. Thus 'good news' seems to explain the observed downward trend in inflation on Figure 1c, for the time period of 2002-2007.

[31] It is argued that China used Mexico as a bridge, given both Mexico WTO and NAFTA memberships to overtake the U.S. textile industry.

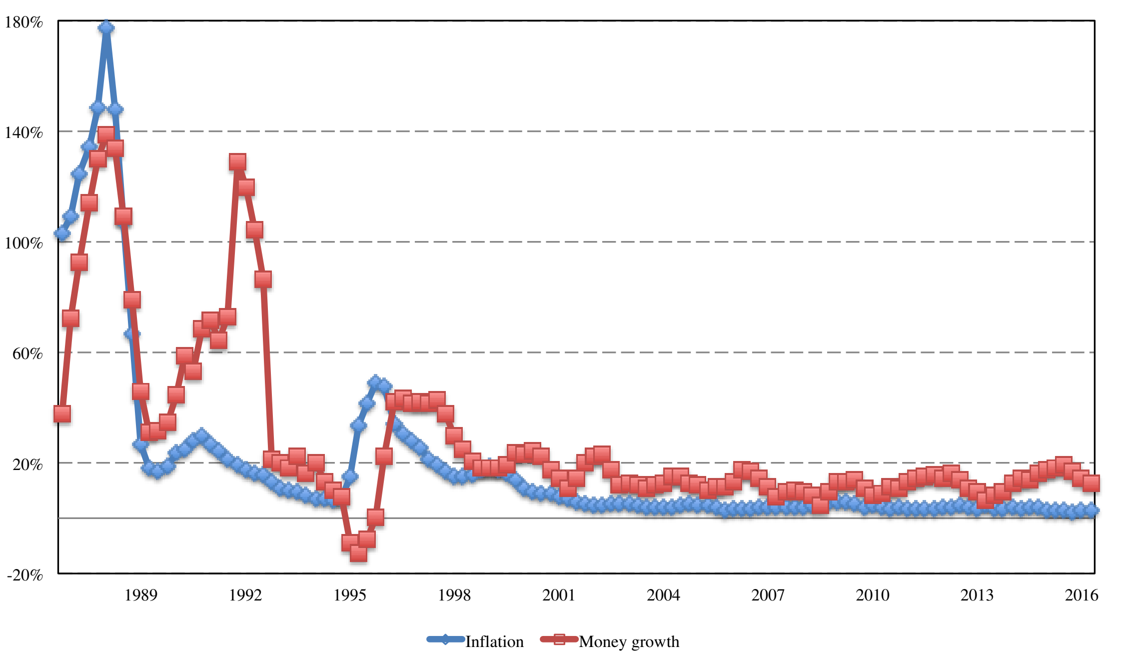
Figure 1d. Mexico: Inflation, output growth. One year percentages 2008-2016 (quarterly data)



Source: Own computations based on INEGI (National Institute of Statistics and Geography).

Figure 1d makes a close-up of Figure 1 for the last time sub-period. This last sub-period goes from 2008 to 2016. It could be observed on Figure 1d, that for the last eight years the 'transition mirror' phenomenon takes place. Inflation and output growth cross at the beginning and ending of 2009. These shifts might reflect U.S. quick economy adjustments *i.e.*, banks bail out implemented through the Emergency Economic Stabilization Act of 2008.

Figure 2. Mexico: Money growth, inflation. One year percentages
1987-2016 (quarterly data)



Source: Own computations based on INEGI (National Institute of Statistics and Geography).

Figure 2 displays almost 30 years of money growth and inflation evolution in Mexico. The inflation movements seem to follow the movements on money growth. Coordination is not perfect, as there is some delay between both monetary macroeconomics indicators.

A lagging inflation seems to be depicted in this figure. Perhaps, this inflation stickiness is linked with the presence of institutional factors *i.e.*, unions.[32] Also, goods price stickiness could counterbalance instantaneous co-movements between money growth and inflation.

As a brief summary of this section, the differences for the summary statistics in each sub-period are evident for the three macroeconomic indicators under study. Hence, it can be inferred from these statistics and the figure analysis, that the Mexican macroeconomic indicators time series are not stationary across time. In addition, the time sub-period of 1994:Q1-2000:Q4 which contains the 'tequila crises' exhibits the greatest variation on all statistics and figures. This heterogeneity between periods is taking into account in the panel VAR presented in next section.

Data sources are reported in Appendix 1. Also, in this appendix, it could be found the leading macroeconomic indicators time availability and units. The Mexican National Institute of Statistic and Geography (*Instituto Nacional de Estadistica y Geografia* INEGI) office frequently updates its time series. In order to keep parsimony with the latest releases and methodologies of this office, only the newest macroeconomic indicators data methodologies are going to be considered in this study.[33] Data transformations are described in Appendix 2.

3. Empirical results

Formal testing of the relationships among the variables described in the introduction and backed by Gillman and Nakov (2004) analytical model takes the following steps. First, the order of integration of the time series and panels are checked to determine which of them could enter in equilibrium paths.[34] As part of these tests the Breakpoint Unit Root Test is included, allowing the possibility of structural breaks in the time series. Next, Granger causality tests are implemented, alongside with structural Breakpoints Identification. These two steps confirm the theoretical conjecture that breaks on the velocity of money correspond to breaks in the relationships among output, inflation and money growth.[35] A quantitative measure of the maxima for these relationships is gauged by the panel VAR dynamic results. Thus, this section presents the panel VAR model and its results alongside with their impulse response functions.

3.1. Econometric model

The use of a panel VAR has several benefits for the empirical analysis. It enlarges the power of statistical tests, and facilitates the analysis of dynamic relationships. This particular lens is explained in Liu and Shumway (2009). These authors also mention that recent developments in time-series econometrics, which combine time-series and cross-sectional information, have provided important possibilities for surmounting low tests power.

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[32] The existence of annual labor contracts do not allow *i.e.*, monthly changes on monetary product-wages.

[33] The *International Financial Statistics* database does not have longer periods than those available at the Mexican National Statistic and Geography Office (INEGI). It has been considered in a first stage of this study, the analysis of output; inflation and money Mexican data considering an old methodology. Since one of the aims of this work is to reproduce Gillman and Nakov econometric (2004) model, which is applied for Hungary and Poland during the 80's, 90's and beginning 00's, using the old methodology for periods previous the 80's, for the case of Mexico, would not add comparative information. This is with respect to contemporaneous economy relationships for the leading macroeconomic indicators under study, in the same guise as in Gillman and Nakov (2004). Thus, given this research scope, the data analysis with an old methodology is left for a future study.

[34] The maxima values of the analytical model (derived from an optimization problem) are analogous to the elasticity coefficients from the panel VAR model (these elasticity coefficients find the optima values of the corresponding distribution). This is it, because both type of models study the same relationships and share the same information and optima. The sojourns taken in both procedures for finding the optima *i.e.*, analytical derivation *vis-á-vis* econometric tests are the corresponding equilibrium paths. In this equilibrium, the optima vales from the analytical and econometric model are the the case. For more information about this lens foundation, see Spanos (2011) regarding theory validation and econometric modelling.

[35] The two steps results are reported in Appendix 3.

In the case of the Mexican economy, the time series for output growth, inflation and money growth are not stationary in levels, not only across all the sampling time, but also between the four selected not overlapping sub-periods, as expressed in section 2. Thus, it is expected a low power on unit root statistics tests as an expression of the time series failure to be stationary and also to the unit root tests failure on detecting structural breaks, or time heterogeneity between and within periods.

Given the Mexican case data characteristics, it is ideal the implementation of panel VAR econometric model to dealt with their great time heterogeneity. Panel VAR considers each sub-period as a cross section, which is deemed necessary for this type of data and econometric model. Panel VAR stacks the selected not overlapping time sub-periods on decreasing chronological order. That it is to say, first the time sub-period of 1987:Q1-1993:Q4; followed by 1994:Q1-2000:Q4; then 2001:Q1-2007:Q4 and finally 2008:Q1-2016:Q1. In this sense, the original ordering of the time series remains unaltered. The advantage of this cross section treatment is that it allows fitting individual elasticity coefficients for each time sub-period. This permits the incorporation of time heterogeneity within and between time sub-periods for output growth; inflation and money growth. It is expected that panel VAR results increase tests power and improve statistic inference.[36] The VAR representation is as follows:

$$\begin{bmatrix}\Delta m_{t,i}\\ \Delta p_{t,i}\\ \Delta y_{t,i}\end{bmatrix} = \sum_{t=1981:Q1}^{t=2016:Q1}\sum_{i=1}^{i=4}\begin{bmatrix}c_{m,i}\\ c_{p,i}\\ c_{y,i}\end{bmatrix} + \sum_{t=1981:Q1}^{t=2016:Q1}\sum_{i=1}^{i=4}\begin{bmatrix}\alpha_{m,i}^{q} & \alpha_{p,i}^{q} & \alpha_{y,i}^{q}\\ \beta_{m,i}^{q} & \beta_{p,i}^{q} & \beta_{y,i}^{q}\\ \gamma_{m,i}^{q} & \gamma_{p,i}^{q} & \gamma_{y,i}^{q}\end{bmatrix}\begin{bmatrix}\Delta m_{t,i-q}\\ \Delta p_{t,i-q}\\ \Delta y_{t,i-q}\end{bmatrix} + \begin{bmatrix}\varepsilon_{m,i}\\ \varepsilon_{p,i}\\ \varepsilon_{y,i}\end{bmatrix}\begin{bmatrix}\Delta m_{t,i}\\ \Delta p_{t,i}\\ \Delta y_{t,i}\end{bmatrix}$$

$$= \sum_{t=1981:Q1}^{t=2016:Q1}\sum_{i=1}^{i=4}\begin{bmatrix}c_{m,i}\\ c_{p,i}\\ c_{y,i}\end{bmatrix} + \sum_{t=1981:Q1}^{t=2016:Q1}\sum_{i=1}^{i=4}\begin{bmatrix}\alpha_{m,i}^{q} & \alpha_{p,i}^{q} & \alpha_{y,i}^{q}\\ \beta_{m,i}^{q} & \beta_{p,i}^{q} & \beta_{y,i}^{q}\\ \gamma_{m,i}^{q} & \gamma_{p,i}^{q} & \gamma_{y,i}^{q}\end{bmatrix}\begin{bmatrix}\Delta m_{t,i-q}\\ \Delta p_{t,i-q}\\ \Delta y_{t,i-q}\end{bmatrix} + \begin{bmatrix}\varepsilon_{m,i}\\ \varepsilon_{p,i}\\ \varepsilon_{y,i}\end{bmatrix}$$

where $\Delta m_{t,i}$ stands for one year percentage logarithm growth in money; t=1987:Q1-2016:Q1 represents within time variation across all the available time period; i=1, 2, 3, 4 stands for between time variation among sub-periods in chronological order (1=1987:Q1-1993:Q4; 2=1994:Q1-2000:Q4; 3=2001:Q1-2007:Q4; 4=2008:Q1-2016:Q1); $\Delta p_{t,i}$ is one year percentage logarithm growth in consumer price index or inflation; $\Delta y_{t,i}$ is the one year percentage logarithm growth in output; $c_{m,i}$ stands for the money growth vector constant; $c_{p,i}$ stands for the inflation vector constant; $c_{y,i}$ stands for the output growth vector constant. The diagonal of the coefficient matrix is read: $\alpha_{m,i}^{q}$ stands for the elasticity coefficient of the money vector with respect to money, where q stands for the number of quarterly lags in the autoregressive terms; $\beta_{p,i}^{q}$ stands for the elasticity coefficient of inflation with respect to inflation; $\gamma_{y,i}^{q}$ stands for the elasticity coefficient of the output growth vector with respect to output;[37] $\varepsilon_{m,i}$ stands for the error term in the money growth vector; $\varepsilon_{p,i}$ stands for the error term in the inflation vector; $\varepsilon_{y,i}$ stands for the error term in the output growth vector.

The dummies variables are frequently used to model structural breaks or to account for between sub-periods time heterogeneity in traditional VAR for time series. In this study, traditional VAR is not longer needed, as panel VAR offers a better approach for the type of data under consideration. As expressed at the beginning of this section, panel VAR accounts for great time heterogeneity (within and between time heterogeneity) of the leading macroeconomic indicators under study, providing high power tests.[38]

[36] 'The low power of traditional test for unit roots in small-and moderate-sized samples can lead to misleading results, but greater power can now be achieved using recent development in panel unit root and cointegration test procedures (Hadri, 2000; Pedroni, 1999)' in Liu *et. al* (2011). Also see Harvey *et al*. (2015).

[37] The rest of coefficients are read in a similar fashion as the elements that conform the diagonal matrix coefficients: rows as independent variables and columns as the dependent ones..

[38] In Appendix A3.1 the unit root tests statistics are reported for Augmented Dickey Fuller (ADF) and Breakpoint Unit Root tests (BURT).

3.2. Panel VAR results

Table 2 below shows that the elasticity coefficients with theoretical and statistic significance are inflation to output growth (-0.031064***) and money to inflation (0.934687***).[39] These results keep a resemblance with the theoretical and statistic significance reported for Hungary and Poland by Gillman and Nakov (2004). For Hungary the corresponding elasticity coefficients are (-1.371***) and (0.139). For Poland these coefficients are (-0.112**) and (0.460**), respectively. Next table 2 reports the results for panel VAR with four quarterly lags.[40]

Table 2. Panel VAR(4) statistics. Mexico. One year percentages changes
1987-2016 (quarterly data) [t-statistics] (lag)

	Output	*Inflation*	*Money*
Output	-0.241155 [-1.84359] (3)	0.08323 [0.35756] (4)	-0.148521 [-1.83341] (3)
Inflation	-0.031064*** [-2.71213] (4)	0.191867*** [7.55019] (4)	-0.000335 [-0.04720] (4)
Money	0.100982 [0.60439] (4)	0.934687*** [2.21619] (1)	-0.040708 [-0.23406] (3)

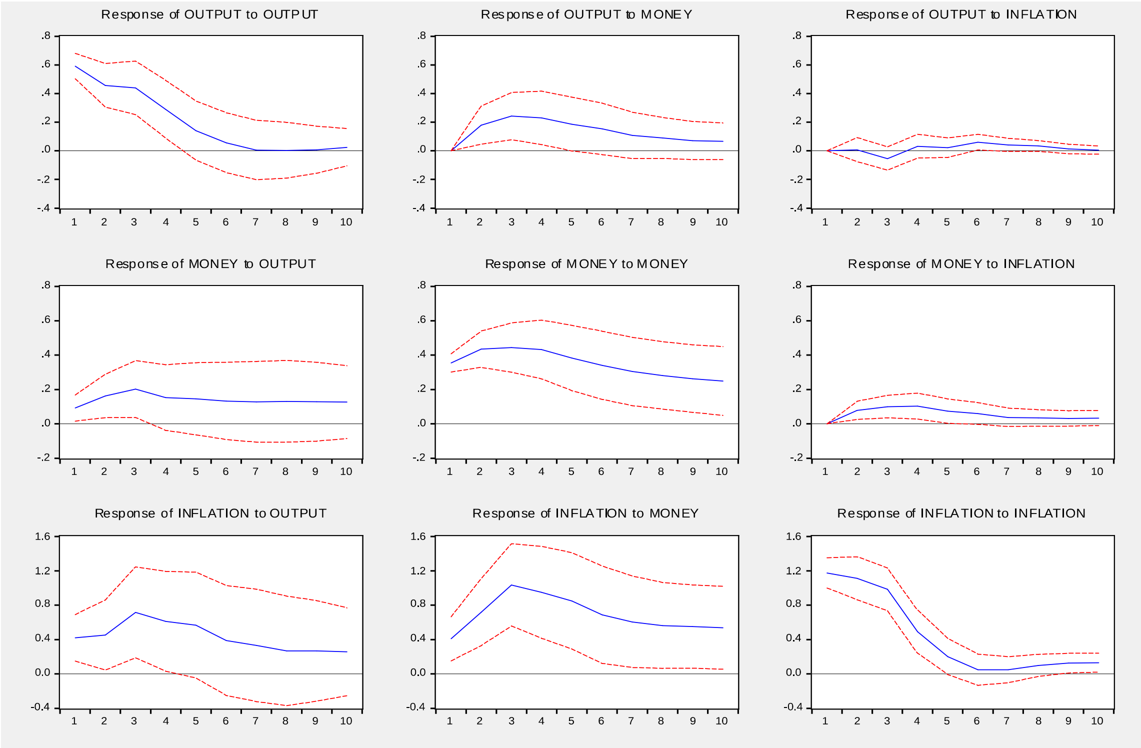
Note 1: Panel VAR(4) stands for a panel vector autoregresive model with four quarterly lags;
Note 2: The constant is not included;
Note 3: * stands for 10%; ** stands for 5%; *** stands for 1% of statistic significance;
Note 4: Lags are selected in terms of their statistic significance.
Source: Own estimates based on INEGI (National Institute of Statistics and Geography), using EViews 9.0.

This resemblance among money growth, inflation and output growth behavior seems to mark something in common among Hungary; Poland and Mexico. Perhaps, these countries share transition stages *i.e.*, changing economic growth model. Other possible shearing feature that these countries may have is that they are classified during the 80, 90 and 00's decades, as middle-income countries. A third candidate as possible explanation for the common macroeconomic behavior is that now economies are more connected, because of computers and international trade chains, which some economist have called an increased in 'globalization.' This last candidate may shed light on why output growth drops in these three countries around 1994 and 2001. Under globalization local instabilities *i.e.*, 'tequila crises' in 1994 and middle U.S. economic recession in 2001, could quickly propagate from one country to another. More research is needed to establish which of these three possible explanations have a higher weight on determining economic growth.

In what follows, it is presented the corresponding impulse-response function (IRF) for panel VAR reported on Table 2.[41]

[39] This last coefficient seems to describe the positive and almost proportional movements of money growth and inflation. For each 1% of money increases, it is expected almost 1% of inflation increases.
[40] For a complete random panel VAR treatment, see Goes (2015) EViews program code.
[41] The IRF are part of a panel VAR lower or upper triangular decomposition *i.e.*, Cholesky decomposition. The basic idea behind this matrix decomposition is to separate forecast from forecasting error. Figure 3 plots the optimal forecast under the assumption that Cholesky-dof adjustement is the correct approach. For the setting of the quadratic loss function and its implication for efficient forecast see Granger and Newbold (1986).

Figure 3: Mexico. Panel VAR, impulse responses functions. Output growth, money growth and inflation. Responses to Cholesky one standard innovations, on a confident interval of two standard errors



Note 1: The response to one standard error innovation is taken asymptotically;
Note 2: The time horizon for impulse responses function is 10 quarters ahead, which is represented by two and a half years.

Source: Own estimates based on INEGI (National Institute of Statistics and Geography), using EViews 9.0.

Figure 3 can be thought as a matrix of 3 by 3. In these terms the diagonal of this matrix is not theoretically relevant, as it plots the impulse response of one macroeconomic indicator in terms of itself. It is worth mentioning again, that the elasticity coefficients that have theoretical and statistic significance, reported on Table 2, are inflation to output growth (-0.031064***) and money to inflation (0.934687***).

The negative effect of inflation to output growth can be visualized in the sub-figure: response of OUTPUT[42] to INFLATION (third column, first row). In here a negative effect is seeing during the first two out of the initial four steps ahead. Afterwards, this effect starts to vanish towards zero as the time horizon increases. The positive effect of money to inflation can be visualized in the sub-figure: response of INFLATION to MONEY (second column, third row). In this case the positive effects are seen during all the 10 steps ahead under consideration.[43]

Conclusion

An attempt has been made to reproduce the analytical and empirical findings of Gillman and Nakov (2004). It is found that the theoretical insights and its transmission mechanism are verified through the connection of velocity of money with the banking sector, output growth and inflation. The analytical model goes beyond the transmission mechanism; as it also explained how inflation causes a decrease in output growth by means of human capital drops.

The empirical findings reported in this research, which uses a panel vector for the case of Mexico are equivalent for a traditional VAR applied for the case of Hungary and Poland. This equivalence reinforces the idea, that under general equilibrium, the analytical and econometric models have to follow similar equilibrium paths.

[42] OUTPUT stands for output growth; MONEY stands for money growth.
[43] The rest of sub-figures are not interpreted as they are not statistical nor theoretically significant.

Basically, the panel VAR elasticity coefficients along with its impulse responses functions show that money determines positively inflation and inflation affects negatively output growth. These empirical facts have being depicted in the figure analysis section: Figure 1a; 1b; 1c and 1d, where the 'transition mirror' effect signals a negative relation between output growth and inflation. Figure 2, for its part displays a positive relation between money and inflation. An attempt to use the narrative approach has being used to explain shifts or structural breaks for the leading macroeconomic indicators under study. That is to say, by matching significant landmarks events with indicators figure crosses. Granger causality tests and structural Breaks Identification support the panel VAR empirical findings, as well as the implications of the analytical model.

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Appendix 1: Data description

Table A1. Data sources

Data ID	Description	Frequency	Time period availability	Source	Units
cp	National consumer price index	monthly	1969-2016	A	2010=100
M1	Nominal	monthly	1985-2016	B	thousands of pesos
Q_{man}	Manufacturing production index	monthly	1980-2008	C	1993=100
Q_{man}	Manufacturing production index	monthly	1993-2016	D	2008=100

Source	Website	Thematic route
A	http://www.inegi.org.mx/sistemas/bie/	Precios e inflación> Índice nacional de precios al consumidor> Mensual> Índice> Índice general y por objeto del gasto> Índice general
B	http://www.inegi.org.mx/sistemas/bie/	Financiero y bursátil> Actividad bancaria> Agregados monetarios> Nueva metodología> M1> Total
C	http://www.inegi.org.mx/sistemas/bie/	Series que ya no se actualizan> Actividad industrial, base 1993> Series originales> Índice de volumen físico de la actividad industrial> Total
D	http://www.inegi.org.mx/sistemas/bie/	Indicadores económicos de coyuntura> Actividad industrial, base 2008> Series originales> Índice de volumen físico> Total de la actividad industrial

Appendix 2: Data transformations

For the case of Mexico, the industrial production index is used to described output growth. The industrial production index is available for two periods: 1980-2008 and 1993-2016. Each of these periods has a different base year: 1993=100 and 2008=100. As these periods overlap during the years from 1993 to 2008, then a homogenous industrial production index can be computed for the time period of 1980 to 2008, with a base year of 2008=100. This industrial production index transformation is the one use in this research.

Monthly data frequency is available for output; price index and money. A transformation in the monthly frequency is needed. This is necessary to exclude monthly seasonal variations. The use of quarterly data frequency is convenient as it excludes these very short variations. In order to compute data with quarterly out of monthly frequency, simple weighted averages are computed as follows:

$$V_q = \frac{\sum_{i=1}^{3} V_{mi}}{3}$$

where V_q stands for quarterly variable (time series); V_{mi} stands for a monthly variable (time series); the sub-index $i = 1$ stands for first month, $i = 2$ stands for second month and $i = 3$ stands for third month.

It is worth to mention, that in the figure analysis section macroeconomic indicators were plotted on year growth percentages based on quarterly data. That it is to say, annual growth rates on a quarterly basis. The corresponding formula for the computation of this type of growth rate is:

$$\Delta V_q = \frac{V_q - V_{q-4}}{V_{q-4}} = \frac{V_q}{V_{q-4}} - 1$$

where ΔV_q stands for a year quarterly growth rate; V_q and V_{q-4} stands for a quarterly variable (time series) with zero and four lags, respectively. And q stands for quarterly frequency.

Appendix 3: Econometric tests

A3.1 Unit root tests

In what follows it is reported the Unit Root tests statistics for the time series under study. The variables output, prices and money are tested in levels and each one of them has its own table. These tests include the Augmented Dickey Fuller (ADF) and the Breakpoint Unit Root Test (BURT). This last test, BURT, provides a formal statistic procedure for estimating time breakpoints.[44]

[44] BURT is built into E-Views 9.0.

Table A3.1.1 Unit Root Test Statistics. Output. Mexico. Selected periods

Subperiod	integration order	Test	Lag*/ Break date	Critical values	Significance**	t-Statistic	Include in test***
				Standard			
1986:Q1-1993:Q4	I(1)	ADF§	0	-3.67017	1% level	-5.971165	A
				-2.963972	5% level	-5.872662	B
				-2.621007	10% level	-5.617793	C
				With break			
	I(1)	BURT§§	2/1992:Q3	-4.949133	1% level	-7.018636	A, D
			0/1989:Q2	-4.443649	5% level	-6.260721	A, E
				-4.193627	10% level		
				Standard			
1994:Q1-2000:Q4	I(1)	ADF§	0	-3.689194	1% level	-5.002118	A
				-2.971853	5% level	-4.942028	B
				-2.625121	10% level	-4.827140	C
				With break			
	I(1)	BURT§§	0/1995:Q2	-4.949133	1% level	-6.041331	A, D
			0/1996:Q1	-4.443649	5% level	-5.290697	A, E
				-4.193627	10% level		
				Standard			
2001:Q1-2007:Q4	I(1)	ADF§	2	-3.689194	1% level	-5.918679	A
				-2.971853	5% level	-7.288389	B
				-2.625121	10% level	-1.957020	C
				With break			
	I(1)	BURT§§	2/2002:Q1	-4.949133	1% level	-8.842373	A, D
			1/2005:Q4	-4.443649	5% level	-9.255249	A, E
				-4.193627	10% level		
				Standard			
2008:Q1-2016:Q1	I(1)	ADF§	4	-3.646342	1% level	-2.854484	A
				-2.954021	5% level	-3.092893	B
				-2.615817	10% level	-2.812956	C
				With break			
	I(1)	BURT§§	5/2010:Q4	-5.347598	1% level	-5.095150	B, D
			6/2010:Q4	-4.859812	5% level	-5.614754	B, E
				-4.607324	10% level		

*: Lag length based on Schwarz information criterion;
**: Critical values for 1% level, 5% level, 10% level of confidence interval;
***: Included in the Augmented Dickey-Fuller test (ADF) equation: A constant; B constant, linear trend and C none;
***: Included in the Breakpoint Unit Root test (BURT) equation: D break type: innovation outlier; E break type: additive outlier;
§: Mackinnon (1996) one-sided p-values, for rejectinf the Null Hypothesis of having a unit root;
§§: Vogelsang (1993) asymptotic one-sided p-values. Break selection: minimize Dickey-Fuller t-statistic.

Source: Own estimates based on *INEGI (National Institute of Statistics and Geography)*, using EViews 9.0.

Table A3.1.2 Unit Root Test Statistics. Inflation. Mexico. Selected periods

Subperiod	integration order	Test	Lags*/ Break date	Critical values	Significance	t-Statistic	Include in test***
				Standard			
1986:Q1-1993:Q4	I(1)	ADF§	4	-3.711457	1% level	-5.258580	A
				-2.981038	5% level	-4.267852	B
				-2.629906	10% level	-2.653780	C
				With break			
	I(1)	BURT§§	5/1991:Q1	-5.347598	1% level	-4.901467	B, D
			7/1990:Q2	-4.859812	5% level	-4.886140	B, E
				-4.607324	10% level		
				Standard			
1994:Q1-2000:Q4	I(1)	ADF§	0	-3.689194	1% level	-3.214339	A
				-2.971853	5% level	-3.088516	B
				-2.625121	10% level	-0.448174	C
				With break			
	I(1)	BURT§§	4/1994:Q4	-4.949133	1% level	-4.968046	A, D
			7/1990:Q2	-4.443649	5% level	-4.247388	A, E
				-4.193627	10% level		
				Standard			
2001:Q1-2007:Q4	I(1)	ADF§	3	-3.689194	1% level	-3.567270	A
				-2.971853	5% level	-9.544680	B
				-2.625121	10% level	-1.408359	C
				With break			
	I(1)	BURT§§	1/2005:Q3	-4.949133	1% level	-10.545210	B, D
			1/2005:Q2	-4.443649	5% level	-12.276540	B, E
				-4.193627	10% level		
				Standard			
2008:Q1-2016:Q1	I(1)	ADF§	1	-3.646342	1% level	-10.763450	A
				-2.954021	5% level	-11.262340	B
				-2.615817	10% level	-0.279684	C
				With break			
	I(1)	BURT§§	1/2009:Q3	-4.949133	1% level	-11.848940	B, D
			1/2013:Q1	-4.443649	5% level	-10.528680	B, E
				-4.193627	10% level		

Note: same footnotes as in Table A3.1.1.

Table A3.1.3 Unit Root Test Statistics. Money. Mexico. Selected periods

Subperiod	integration order	Test	Lags*/ Break date	Critical values	Significance	t-Statistic	Include in test***
				Standard			
1986:Q1-1993:Q4	I(1)	ADF[$]	0	-3.67017	1% level	-5.248626	A
				-2.963972	5% level	-5.860554	B
				-2.621007	10% level	-0.801068	C
				With break			
	I(1)	BURT[$$]	0/1990:Q3	-4.949133	1% level	-6.146614	A, D
			7/1990:Q1	-4.443649	5% level	-9.927883	A, E
				-4.193627	10% level		
				Standard			
1994:Q1-2000:Q4	n.a.	ADF[$]			1% level		A
					5% level		B
					10% level		C
				With break			
	I(1)	BURT[$$]	6/1997:Q1	-4.949133	1% level	-5.319983	A, D
			6/1997:Q1	-4.443649	5% level	-4.831940	A, E
				-4.193627	10% level		
				Standard			
2001:Q1-2007:Q4	I(1)	ADF[$]	4	-3.689194	1% level	-2.862358	A
				-2.971853	5% level	-4.406145	B
				-2.625121	10% level	-0.096666	C
				With break			
	I(1)	BURT[$$]	5/2005:Q4	-4.949133	1% level	-4.686780	A, D
			5/2005:Q2	-4.443649	5% level	-4.383400	A, E
				-4.193627	10% level		
				Standard			
2008:Q1-2016:Q1	n.a.	ADF[$]			1% level		A
					5% level		B
					10% level		C
				With break			
	I(1)	BURT[$$]	5/2014:Q3	-5.347598	1% level	-4.932843	B, D
			1/2013:Q1	-4.859812	5% level	-7.460564	B, E
				-4.607324	10% level		

Note: same footnotes as in Table A3.1.1.

As it can be seen in Tables A.3.1.1-3 output, prices and money are stationary in first differences I(1). Next, it is presented in Table A.3.1.4 the unit root test for the velocity of money. BURT identifies its structural breaks.

Table A3.1.4 Unit Root Test Statistics. Velocity. Mexico. Selected periods

Subperiod	integration order	Test	Lags*/ Break date	Critical values	Significance	t-Statistic	Include in test***
				Standard			
1986:Q1-1993:Q4	I(1)	ADF[$]	0	-3.72407	1% level	-3.413429	A
				-2.986225	5% level	-3.991652	B
				-2.632604	10% level	-3.401622	C
				With break			
	I(1)	BURT[$$]		-4.949133	1% level	n.a.	
			3/1990:Q1	-4.443649	5% level	-4.860635	A, E
				-4.193627	10% level		
				Standard			
1994:Q1-2000:Q4	I(1)	ADF[$]	0	-3.689194	1% level	-4.831498	A
				-2.971853	5% level	-4.699982	B
				-2.625121	10% level	-4.932212	C
				With break			
	I(1)	BURT[$$]		-4.949133	1% level	n.a.	
			6/1996:Q3	-4.443649	5% level	-5.099792	A, E
				-4.193627	10% level		
				Standard			
2001:Q1-2007:Q4	I(1)	ADF[$]	3	-3.689194	1% level	-4.723646	A
				-2.971853	5% level	-4.636183	B
				-2.625121	10% level	-6.276820	C
				With break			
	I(1)	BURT[$$]		-4.949133	1% level	n.a.	
			0/2006:Q2	-4.443649	5% level	-6.499935	A, E
				-4.193627	10% level		
				Standard			
2008:Q1-2016:Q1	I(1)	ADF[$]	0	-3.646342	1% level	-3.387331	A
				-2.954021	5% level	-3.324556	B
				-2.615817	10% level	-3.439135	C
				With break			
	I(1)	BURT[$$]		-4.949133	1% level	n.a.	
			1/2013:Q1	-4.443649	5% level	-4.233345	A, E
				-4.193627	10% level		

Note: same footnotes as in Table A3.1.1.

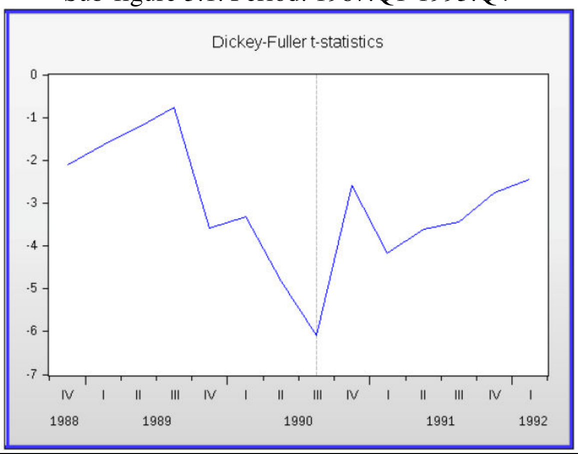
Not surprisingly, BURT structural breaks identification follows the velocity of money breaks. This pattern is described in Gillman and Nakov (2004). That it is to say, structural breaks in the velocity of money determines structural breaks in output growth and inflation. These links are described in Table A3.1.5.

For the case of Mexico the traditional VAR is not needed, neither dummies for modelling the structural breaks. This is because panel VAR results have a higher test power and improve statistical inference. However, the structural breaks identification through the velocity of money is worth mentioning, as it establishes the transmission mechanism, between the analytical model of section 1 and the leading macroeconomic indicators under study. Next, the structural breaks figures detected with BURT for velocity of money are reported:

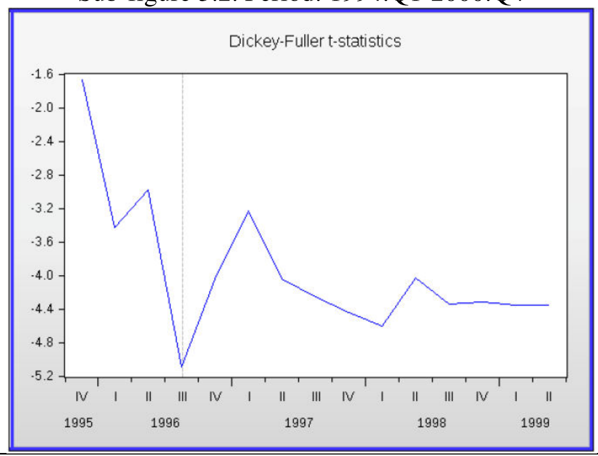
Figure 3: Mexico: Velocity of money. One year percentages 1987-2016

Selected periods (quarterly data)

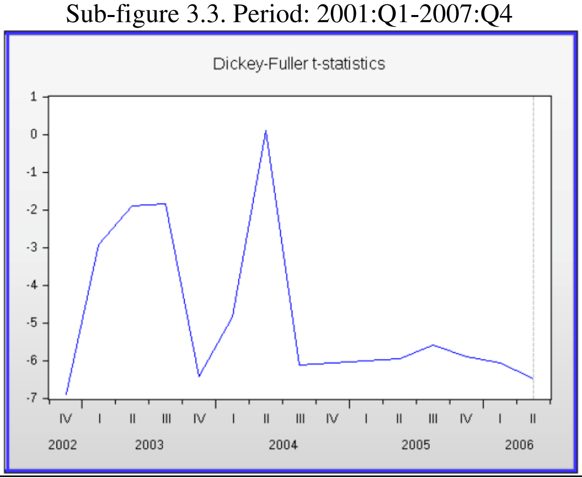
Sub-figure 3.1. Period: 1987:Q1-1993:Q4



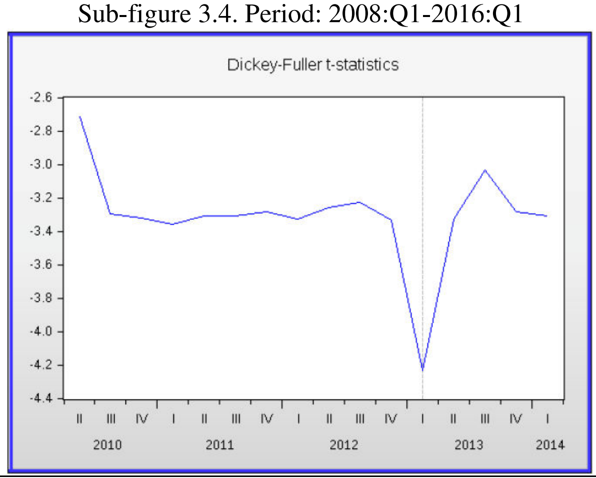
Sub-figure 3.2. Period: 1994:Q1-2000:Q4



Sub-figure 3.3. Period: 2001:Q1-2007:Q4



Sub-figure 3.4. Period: 2008:Q1-2016:Q1



Source: Own computations based on INEGI (National Institute of Statistics and Geography), using EViews 9.0.

Figure 3 is a visual representation of the velocity of money structural breaks reported on Table A3.1.4. The vertical line in sub-figure 3.1 marks the break date in 1990:Q1. Basically, this line cuts the Dickey-Fuller t-statistics time series in its minimum point. A similar interpretation can be applied for the vertical lines on sub-figures 3.2; 3.3 and 3.4. The corresponding structural break dates are 1996:Q3; 2006:Q2 and 2013:Q1.

In addition, if the number of lags reported on BURT for the velocity of money are applied to the structural breaks for the chosen macroeconomic indicators, then it can be seen on Table A3.1.5, *i.e.*, Block 1 almost a pair wise match, among the structural breaks for money velocity and the time series crosses depicted on Figures 1a; 1b; 1c, and 1d. In what follows, Table A3.1.5 also reports on Block 2; Block 3 and Block 4 the relationships among velocity of money and the corresponding leading macroeconomic indicators. The structure of Table A3.1.5 is a bipartite network directed with a mixed approach, applied to these macroeconomic indicators structural breaks identification.

Table A3.1.5 Breakpoints identification, velocity of money, money, inflation and output growth. Mexico. Selected periods

Block 1

BURT	Velocity of money	Lags	Figures time series crosses	output growth and inflation
1986:Q1-1993:Q4	**3/1990:Q1**	3 lags to velocity give 1989 →	1986:Q1-1993:Q4	**1989**
				1993
1994:Q1-2000:Q4	**6/1996:Q3**	6 lags to velocity give 1995 →	1994:Q1-2000:Q4	**1995**
				1996
2001:Q1-2007:Q4	0/2006:Q2		2001:Q1-2007:Q4	2002
				2003
				2005
2008:Q1-2016:Q1	**1/2013:Q1**		2008:Q1-2016:Q1	2008
		1 lag to velocity gives 2013 →		**2013**

Block 2

BURT	Inflation	Lags	BURT	Velocity
1986:Q1-1993:Q4	**5/1991:Q1**	5 lags to inflation give 1990 →	1986:Q1-1993:Q4	**3/1990:Q1**
	7/1990:Q2			
1994:Q1-2000:Q4	**4/1994:Q4**	← 6 lags to inflation give 1994	1994:Q1-2000:Q4	**6/1996:Q3**
	7/1990:Q2			
2001:Q1-2007:Q4	1/2005:Q3		2001:Q1-2007:Q4	0/2006:Q2
	1/2005:Q2			
2008:Q1-2016:Q1	1/2009:Q3		2008:Q1-2016:Q1	
	1/2013:Q1	← 1 lag gives 2013 →		**1/2013:Q1**

Block 3

BURT	Money	Lags	BURT	Velocity
1986:Q1-1993:Q4	0/1990:Q3		1986:Q1-1993:Q4	3/1990:Q1
	7/1990:Q1			
1994:Q1-2000:Q4	**6/1997:Q1**	6 lags to money give 1996 →	1994:Q1-2000:Q4	**6/1996:Q3**
	6/1997:Q1			
2001:Q1-2007:Q4	5/2005:Q4		2001:Q1-2007:Q4	0/2006:Q2
	5/2005:Q2			
2008:Q1-2016:Q1	5/2014:Q3	5 lags to money give 2013 →	2008:Q1-2016:Q1	**1/2013:Q1**
	1/2013:Q1	1 lag gives 2013		

Block 4

BURT	Output	Lags	BURT	Velocity
1986:Q1-1993:Q4	**0/1989:Q2**	← 3 lags to velocity gives 1989	1986:Q1-1993:Q4	**3/1990:Q1**
	2/1992:Q3			
1994:Q1-2000:Q4	**0/1995:Q2**	← 6 lags to velocity gives 1995	1994:Q1-2000:Q4	**6/1996:Q3**
	0/1996:Q1			
2001:Q1-2007:Q4	2/2002:Q1		2001:Q1-2007:Q4	
	1/2005:Q4			0/2006:Q2
2008:Q1-2016:Q1	5/2010:Q4		2008:Q1-2016:Q1	1/2013:Q1
	6/2010:Q4			

Note 1: BURT stands for Breakpoint Unit Root test;
Note 2: Breakpoint dates matches are in boxes. Arrows indicates lag directionality.
Source: Own computations based on INEGI (National Institute of Statistics and Geography) and Barabasi (2014, p.18).

Table A3.1.5 synthetizes the structural breaks information gather through BURT and the figure analysis of section 2. The break identification of the velocity of money over the relevant macroeconomic indicators seems a first look a complicated task. However, the network approach simplifies this work.

On Block 1 all its arrows go from left to right. These indicate that for three sub-periods velocity of money structural breaks determines the time series of output growth and inflation crosses depicted on section 2: figures 1a; 1b; 1c and 1d. On Block 2, velocity of money determines inflation for two sub-periods: 1994:Q1-2000:Q4 and 2008:Q1-2016:Q1.

On Block 3 money determines velocity for two sub-periods: 1994:Q1-2000:Q4 and 2008:Q1-2016. This last information conforms to the analytical model insights on Gillman and Nakov (2004), where bank sector productivity in terms of money supply determines the velocity of money. Block 4 displays that velocity of money determines output growth in two sub-periods: 1986:Q1-1993:Q4 and 1994:Q1-2000:Q4. This influence of velocity of money for these periods on output growth follows Gillman and Nakov (2004) analytical model, where the effect of velocity of money is translated into output through the inflation tax.

In what follows is presented the Unit Root tests for panel time series in levels for output; inflation and money. Here, the integration order for the three leading macroeconomic indicators under analysis is one.

Table A3.1.6 Panel Unit Root Test Statistics. Output. Mexico. Selected periods

Cross-section included	integration order	Test	Lag*	Pooled cross section statistic	Prob.**	Include in test***
			Output			
1986:Q1-1993:Q4 1994:Q1-2000:Q4 2001:Q1-2007:Q4 2008:Q1-2016:Q1	I(1)	LLC	0 to 3 0 to 3 0 to 3	-7.05030 -2.11507 -7.59052	0.0000 0.0172 0.0000	A B C
			Inflation			
1986:Q1-1993:Q4 1994:Q1-2000:Q4 2001:Q1-2007:Q4 2008:Q1-2016:Q1	I(1)	LLC	0 to 3 0 to 3 0 to 7	-5.74804 -7.49975 -2.70969	0.0000 0.0000 0.0034	A B C
			Money			
1986:Q1-1993:Q4 1994:Q1-2000:Q4 2001:Q1-2007:Q4 2008:Q1-2016:Q1	I(1)	LLC****	0 to 3 0 to 3 0 to 3	-2.05195 -1.54285 0.35902	0.0201 0.0614 0.6402	A B C

*: Lag length based on Schwarz information criterion;
**: Probabilities are computed assuming asymptotic normality;
***: Included in the Common Unit Root-Levin, Lin, Chu test (LLC) equation: A constant; B constant and linear trend and C none;
****: Newey-West automatic bandwidth selection and Parzen kernel.

Source: Own computations based on INEGI (National Institute of Statistics and Geography), using EViews 9.0.

A3.2 Granger causality tests

The Granger causality panel representation for the macroeconomic indicators under analysis is:

$$\Delta \log output_t = \beta_{1,1}\Delta \log output_{t-k} + \beta_{1,2}\Delta \log inflation_{t-k} + \beta_{1,3}\Delta \log money_{t-k} + \varepsilon_t$$

$$\Delta \log inflation_t = \beta_{2,1}\Delta \log output_{t-k} + \beta_{2,2}\Delta \log inflation_{t-k} + \beta_{2,3}\Delta \log money_{t-k} + \varepsilon_t$$

$$\Delta \log money_t = \beta_{3,1}\Delta \log output_{t-k} + \beta_{3,2}\Delta \log inflation_{t-k} + \beta_{3,3}\Delta \log money_{t-k} + \varepsilon_t$$

where Δ stands for first differences; log stands for logarithm; i=1, 2, 3, 4 stands for the number of cross sections, indicating sub-periods in chronological order: 1=1987:Q1-1993:Q4; 2=1994:Q1-2000:Q4; 3=2001:Q1-2007:Q4; 4=2008:Q1-2016:Q1; $\beta_{j,l}$ is the respective elasticity coefficient, with j stands for the row number and l stands for the column number; t is time in quarters and k is time lags; ε_t stands for the error term.

The Granger causality test reported on Table A3.2.1 is based on Dumitrescu-Hurlin (2012), where the betas elasticity coefficients are allowed to differ in each time sub-period. One step of this test is computed by running a standard Granger causality regression in each sub-period. It is important to mention that Granger causality tests for panel statistics is a Zbar, which normalizes the usual t-student statistic reported on time series Granger causality tests. This normalization happens across sub-periods.

Table A3.2.1 Leading macroeconomic indicators. Pairwise Granger Causality Tests. Mexico. 1986:Q1-2016:Q1

Dumitrescu Hurlin (individual coefficients)

Null hypothesis		2*	3*	4*	5*
money growth does not Granger cause inflation	Zbar-Stat. Prob.	2.44034*** 0.0147	1.65725* 0.0975	1.57593* 0.1150	0.25793*** 8.E-01
inflation does not Granger cause output growth	Zbar-Stat. Prob.	7.14614*** 9.E-13	6.46391*** 1.E-10	5.65114*** 2.E-08	2.42348*** 0.0154
		i		i	
output growth does not Granger cause money growth	Zbar-Stat. Prob.	1.0795 0.2804	0.2912 0.7709	0.9105 0.3626	0.0208 0.9834

Note 1: All variables are stationary in first differences of logarithms;
Note 2: When the Prob. is less than 0.1, then Zbar-Stat is statistical significant at a 10%. Similarly for 0.05 (5%) and 0.01 (1%);
Note 3: * Lags length measured in quarters;
Note 4: The null hypothesis is rejected when Zbar-Stat. is statistic significant.

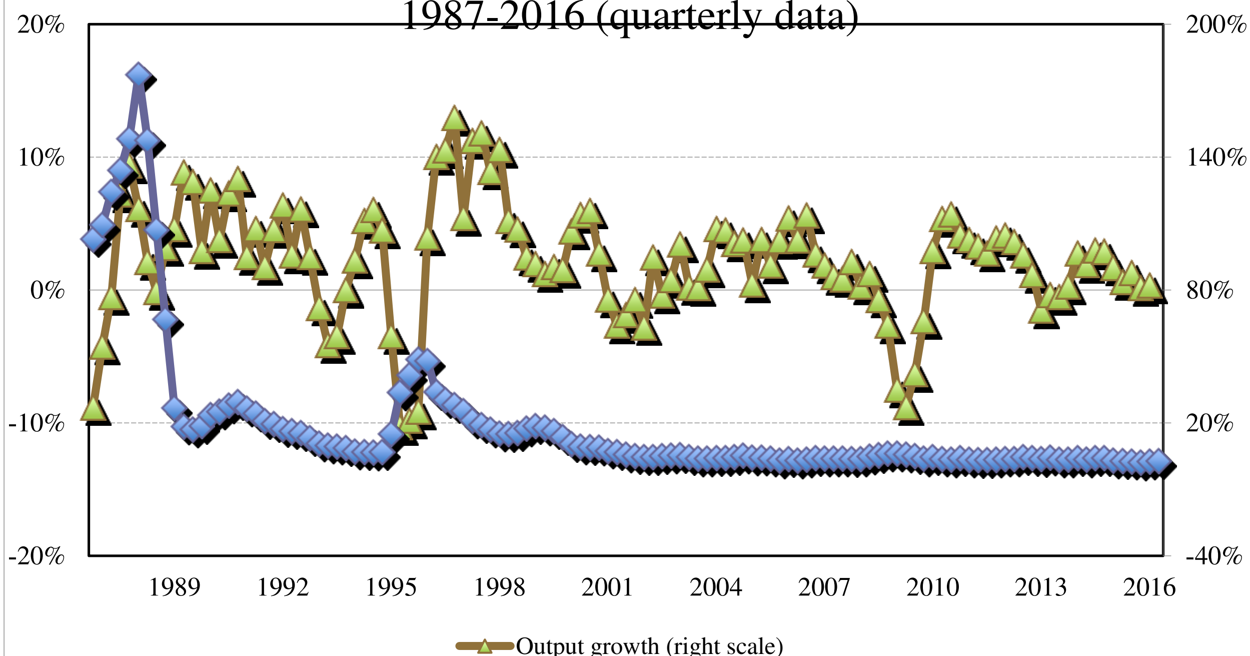
Source: Own computations based on INEGI (National Institute of Statistics and Geography), using Eviews 9.0.

On one hand, it is found in Table A3.2.1 that money causes inflation for 2, 3, 4 and 5 quarters lags. This is it, because the null hypothesis of not Granger causality can be rejected at a 10% and 5% of statistic significance. In the same fashion, inflation Granger causes output growth for lags 2; 3; 4 and 5 with a significance of 1%. On the other hand, output growth is found not to Granger cause money. For output lags 2 and 4 a column sub-heading of i is reported in Table A3.2.1. This sub-heading indicates that for these lags, output growth also does not Granger cause inflation. The letter i on the table indicates the first letter of inflation.

Not surprisingly, Granger causality tests reported on Table A3.2.1 are aligned with Granger causality test results reported on Gillman and Nakov (2004), for the case of Hungary and Poland. A summary of Gillman and Nakov (2004) of Granger causality is as follows: money Granger causes inflation; inflation Granger causes output growth; and output growth does not Granger cause money and inflation.

Figure 1. Mexico: Inflation, output growth. One year percentages

1987-2016 (quarterly data)



Source: Own computations based on INEGI (National Institute of Statistics and Geography).