

Curvature Loading on Term Structure of Interest Rate: Indonesia Bond Market Context

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Abstract

The term structure of interest rate model in Indonesia is based on the Svensson (1994) model development which has accepted globally and being used by most countries. However, the model has tendency to be accurate in short- and medium-term but less accurate in longer term due to low population of bonds in long term maturity. The research of Diebold and Li in 2006 overcomes this situation with their simplification into three factors (level, slope and curvature) and one curvature loading. They suggested the value of curvature loading to be fixed in value 0.0609 which based on US bond market. The following research of Diebold, Rudebusch and Aruoba (DRA) in 2007 introduce the model into state-space representation. With the DRA model, the authors could find the new curvature loading into country specific bond market. With the curvature that is based on specific country, the calculation of the term structure would be more applicable. Therefore, the authors propose to use their method to find the alternative curvature loading for Indonesia bond market context.

Keywords: loading curvature parameter, term structure of interest rate, Diebold-Li model, government bond, Indonesia

1. Introduction

Term structure of interest rate or yield curve plays an important role and an integral part of Indonesia bond market. The bond market is main tool for government to cover the budget deficit which is funded by the issuance debt securities both in domestic and foreign currency (Cociuba, Shukayev, & Ueberfeldt, 2019). As per 2019, total Indonesia government debt securities are IDR 3,874 trillion with 74% are in Rupiah denominated currency (Kemenkeu, 2019). The term structure was developed from several method; such as: the Spline method, non-arbitrage method and parametric method. Development of the last method is mainly based on research by Nelson Siegel in 1987 (Nelson & Siegel, 1987). The Nelson Siegel based family is the most common model used in reconstructing the yield curve that use by central banks especially in the developed market such as central banks of European Central Bank, South Korea, Belgium, Finland, France, Germany, Italy, Norway, Spain, and Switzerland (BIS, 2005). One of the most acknowledged method from Nelson Siegel family is the Svensson model.

This model is already accepted worldwide as the standard model for Central Banks and Bond Agencies to develop their country yield curve, and the yield curve itself is the main component to provide bond price fair value. However, the Svensson model has tendency to have great accuracy on short- and medium-term bond, but is less accurate in longer term since it depends on the population of bonds in each term. Most of liquid bond (ie benchmark bonds) are clustered in between three until ten years of maturity. Therefore, in shorter term, the original Svensson model can give better predictions for fair value pricing. However, for longer term more than 10 years of maturity, it has tendency that lesser series of bond published by the Government and therefore there are lower populations. Since the population is less and lesser trade activity in the market, the creation of bond yield curve would become less accurate (Dajcman, 2015).

In Indonesia, the population of government bonds are grouped into short and medium maturity thus it has less series in long term maturity. This is an indicator that in general, the term structure in Indonesia is not intact.

Hence, the theory behind the term structure has more concern in arbitrage-free where the effort is to fit the term structure at point in time, as opposed to dynamic model. On the other hand, the affine equilibrium term structure has more concern in the dynamic in the short rate and so potentially applicable to forecasting, but again the focus only on the in-sample fit as opposed to out-sample forecasting. Further development tried to overcome those situations, Nelson Siegel was able to model forward rate directly with three latent factors (level, slope and curvature) of the term structure. Unlike the affine model, the Nelson Siegel model greatly improved the forecasting performance across maturities and has become very popular among Central Banks.

Term structure construction in Indonesia is also using the Nelson Siegel family that is based on Svensson method which the theory was established in 1994. The yield curve contains information about the yield of zero-coupon bonds of various maturities at certain date (Alfeus, Grasselli, & Schlogl, 2020). The process of constructing yield curve is a not straight forward due to limitation on the zero-coupon bonds on the market which are the essential ingredient of yield curve. In fact, majority of bonds traded in the market bear coupons. The yield to maturity on coupon bonds in which has different maturities and coupons are not directly comparable to each other. As a result, a method that uniform is needed in order to obtain yield curve (Fisher, Nychka, & Zervos, 1995).

Currently, the development of Nelson Siegel is based on Diebold and Li research in 2006. It simplifies the model into three parameters and one loading parameter on curvature. Thus, this paper is focus on the estimation of curvature loading of term structure of interest rate as the alternative of loading that was determined by Diebold and Li in 2006. In this research, the authors would like to observe and propose the loading alternative that is based on Indonesia bond market and therefore could contribute on development of overall bond market. The alternative of curvature loading is an important part of term structure of interest rate development, with the more accurate loading it would provide term structure that is more in line with market movement.

2. Theoretical Framework

Diebold Li model as the recent development of Nelson Siegel family model provides a simple methodology of estimation of yield curve that is adjusting to market observation and put concentration in the instantaneous forward rate. The main purpose is to get better fit on the yield curve, especially in the longer term of the curve. It was obtained by reparametrizing the original formulation the yield $y_m(\tau_1)$ as a function of three parameters (Level or long-term factor, Slope or short-term factor and Curvature or medium-term factor) and one λ that determines the maturity at which the loading of Curvature will be maximized. Under the original Diebold Li model, the λ set fixed at value 0.0609 thus the process could be linear. The equation of spot rate is as below:

$$S_m(\tau_1) = L_t + S_t \left(\frac{1 - e^{-\lambda\tau_1}}{\lambda\tau_1} \right) + C_t \left(\frac{1 - e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} \right) + \varepsilon_{i,m}$$

Where:

S_m = denotes the set of spot yield curve

τ_1 = corresponding maturity

L_t = level

S_t = slope

C_t = curvature

λ = lambda

The state space model or SSM and Kalman filter is an important step in the Diebold Li model in order to incorporate time-varying lambda. A state space model is formulated in discrete time using a multivariate difference equation or a multivariate differential equation in continuous time. This process describes the dynamic of the state vector X_t and a static relation between the state vector and the multivariate observation Y_t . Therefore, a linear state space model consist of two sets equations as follows (Diebold & Li, Forecasting the Term Structure of Government Bond Yields, 2006; Diebold, Rudebusch, & Aruoba, The Macroeconomy and The Yield Curve: A Dynamic Latent Factor Approach., 2007):

$$\text{System equation} \quad X_t = A_t X_{t-1} + B_t u_{t-1} + e_{1,t}$$

$$\text{Observation equation} \quad Y_t = C X_t + e_{2,t}$$

where X_t is the N-dimensional random state vector that is not directly observable, the u_t is the deterministic input vector, and Y_t is the vector of observable stochastic output. The A_t , B_t and C_t are the deterministic matrices in which the parameters are embedded.

Lastly, the uncorrelated white noise processes are represented by $e_{1,t}$ and $e_{2,t}$. With above equations, the system equation X_t describes the evolution of the system states and the observation equation Y_t describes what can be directly measured.

Kalman filter was found by Rudolf E Kalman, he developed a theory in statistic and control also known as linear quadratic estimation. The filter is an algorithm that use a series of measurements observed over time that contains statistical noise and other inaccuracies and produces estimates of unknown variables that to tend to be more accurate by estimating a joint probability distribution over the variables for each timeframe. The filter has numerous applications mainly in Engineering (control, aviation, navigation, and etc). However, due to the same concept of disturbance, Kalman filter also widely applicable in economic time series analysis.

The filter works in two-step process: prediction step and updated step. In the prediction step, Kalman filter produces estimates of the current state variables along with their uncertainties. Once the outcome of next measurement is observed, next step is update step. The algorithm is recursive, so it can run in real time or only using the present input and previously calculated state and its uncertainty matrix thus no additional past information is required.

3. Data

The yield curve data consist of time series data of 12 years period of daily data Indonesia bonds for the maturities 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29 and 30 years. The yields of each data points are recorded in percent from January 2007 until January 2020. There are 2946 trading days and 88,380 yield data are being processed under the research as shown in Table 1. The data is populated from Indonesia Ministry of Finance that is recorded in Bloomberg terminal. During 12 years of trading activities, it has through several downtrend, sideways and downtrend cycle therefore this research has covered various conditions in Indonesia bond market.

Table 1. Indonesia Yield Curve 2007 - 2020

Maturities	Mean	Median	St Deviation
1Y	6.78%	6.68%	1.78%
2Y	7.26%	7.22%	1.85%
3Y	7.52%	7.42%	1.95%
4Y	7.71%	7.56%	2.00%
5Y	7.82%	7.64%	2.03%
6Y	8.01%	7.80%	2.00%
7Y	8.13%	7.94%	1.98%
8Y	8.22%	7.99%	1.99%
9Y	8.29%	8.02%	2.02%
10Y	8.31%	7.97%	2.06%
11Y	8.46%	8.15%	1.97%
12Y	8.61%	8.27%	1.98%
13Y	8.75%	8.38%	2.01%
14Y	8.74%	8.37%	2.02%
15Y	8.75%	8.35%	2.03%
16Y	8.88%	8.50%	1.99%
17Y	8.92%	8.50%	1.99%
18Y	8.98%	8.56%	2.02%
19Y	8.99%	8.53%	2.01%
20Y	8.97%	8.54%	1.99%
21Y	9.01%	8.61%	1.98%
22Y	9.06%	8.68%	1.98%
23Y	9.10%	8.71%	1.96%
24Y	9.11%	8.73%	1.97%
25Y	9.12%	8.73%	1.98%
26Y	9.13%	8.74%	1.98%
27Y	9.14%	8.77%	1.99%
28Y	9.15%	8.78%	2.00%
29Y	9.16%	8.80%	2.01%
30Y	9.18%	8.81%	2.01%

In this research, the authors would like to process term structures that are based on original Diebold Li model and alternative curvature loading.

The original Diebold Li model is estimated using two-step approach that was determined under their research. The approach is as follows:

1. First step is fixing λ at 0.0609 which implies the loading curvature (or medium term) is maximized at 30 months. It equates three factors (level, slope and curvature) to the regression coefficient by Ordinary Least Square (OLS) and accumulates time series from estimated factors of level, slope and curvature.
2. Second step is to fit the time series from first step into first-order autoregressive model.

The alternative curvature loading is estimated using the State-Space model (SSM). This method was introduced by Diebold, Rudebusch and Aruoba in 2007. The maximum likelihood estimation (MLE) of the model is processed with Kalman filter and 3x3 AR coefficient matrix.

4. Results

Result of original curvature loading (two-step) and alternative curvature loading are shown in Figure 1 as for the level or long-term factor, Figure 2 as for the slope or short-term factor and Figure 3 as for the curvature or medium-term factor. The result from Figure 4 shown the estimated alternate curvature is maximized at 0.0060266 value in comparison with the original loading at 0.0609.

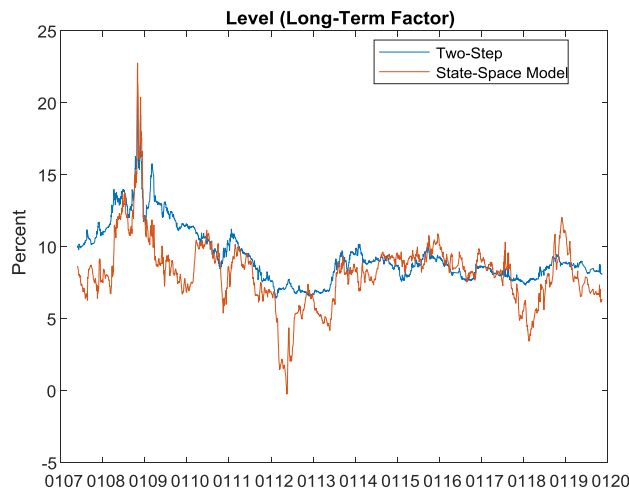


Figure 1. Level (long term factor) of original loading (two-step) and alternative loading (state-space)

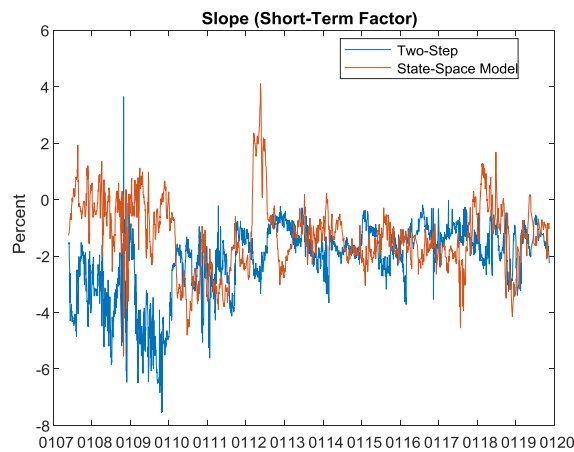


Figure 2. Slope (short term factor) of original loading (two-step) and alternative loading (state-space)

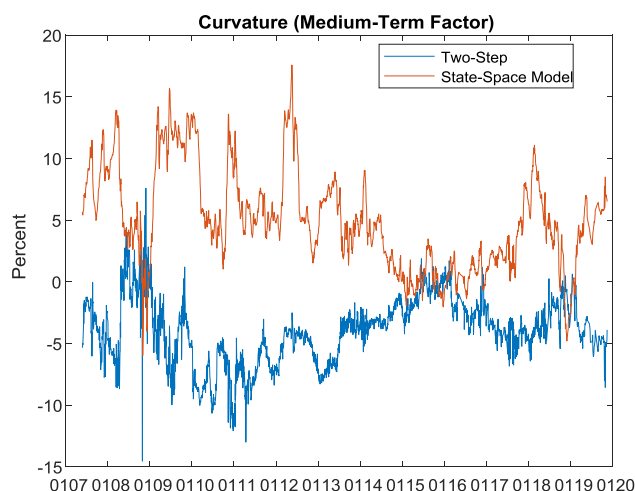


Figure 3. Curvature (medium term factor) of original loading (two-step) and alternative loading (state-space)

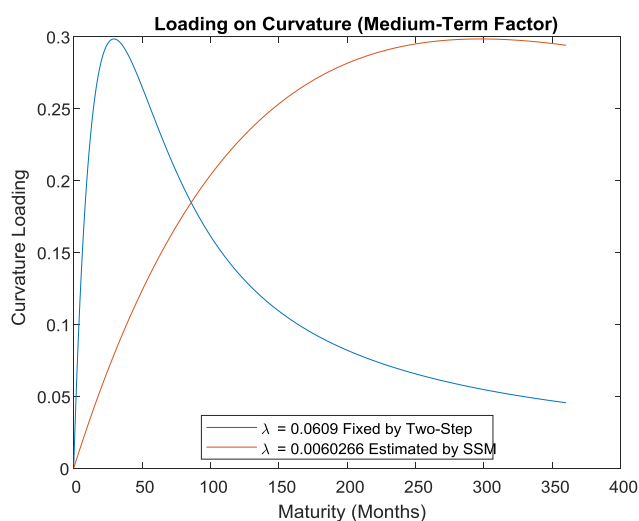


Figure 4. Curvature loading of original loading (two-step) and alternative loading (state-space)

The blue colour graphic in Figure 1, 2 and 3 are showing level, slope and curvature under the two-step process that based on fixed lambda or curvature loading value at 0.0609. While the red colour graphic in Figure 1, 2 and 3 are showing level, slope and curvature under the state-space process that based on the alternative curvature loading. Figure 4 is showing the curvature loading from both two-step and state-space process. Each result of level, slope, curvature and loading from Figure 1, 2, 3 and 4 are important input for spot rate equations under Diebold-Li model that would provide the yield level of each maturity dates. Those yields are further constructed into the term structure date of interest rate for each particular dates.

5. Conclusions

This paper presented on obtaining a term structure of interest rate components for Indonesia bond market under two method of term structure building. The first method is based on Diebold Li model established in 2006 which proposed a reparameterization of Nelson Siegel model in 1987 in simple three factors level, slope and curvature. The first method provided a static curvature loading at value of 0.0609 that is based on US bond market. And second method is based on Diebold, Rudebusch and Aruoba in 2007 which proposed a new value of curvature loading that could be localized on specific country.

Value of 0.0060266 as the curvature loading for Indonesia context is the result of 12 years of Indonesia bond market yield curves. The period of research has included the period of 2007-2008 crisis in financial market, market recovery and market expansions until 2020. The authors believe the value of curvature loading in Indonesia context would contribute on study development of term structure of interest rate.

Further research should investigate if the effects of macroeconomic phenomena such as monetary policy, inflation expectations, and real business activity differ according to the particular interest rate regime realized for the term structure, for example in periods of low interest rates, monetary policy and real business activity have a greater or smaller effect on the longer maturities of the yield curve, vice versa. Finally, additional analysis can be further developed to be more detailed on several timeframe such as period of crisis, period of recovery and period of economic expansion. Additional analysis in bond market in other countries also would provide beneficial development in the study of term structure of interest rate.

References

- Aazim, M. (2010). Monetary Policy Effectiveness & Yield Curve Dynamics - US Experience from A Heterogeneous Economic Perspective (2000-2009). *Journal of Academic Research in Economics*, 2(3), 297-310.
- Alfeus, M., Grasselli, M., & Schlogl, E. (2020). A consistent stochastic model of the term structure of interest rates for multiple tenors. *Journal of Economic Dynamics and Control*, 114(5).
- BIS. (2005). Zero-Coupon Yield Curves: Technical Documentation. *BIS Papers*, 25, 1-37.
- Cociuba, S. E., Shukayev, M., & Ueberfeldt, A. (2019). Managing Risk Taking With Interest Rate Policy and Macroprudential Regulations. *Economic Inquiry*, 57(2), 1056-1081.
- Dajcman, D. (2015). An Empirical Investigation of the Nexus between Sovereign Bonds Yields and Stock Market Returns. *Inzinerine Ekonomika-Engineering Economics*, 26(2), 108-117.
- Date, P. (2011). Linear and non-linear filtering in mathematical finance: a review. *IMA Journal of Management Mathematics*, 22, 195-211.
- Diebold, F. X., & Li, C. (2006). Forecasting the Term Structure of Government Bond Yields. *Journal of Econometrics*, 130(2), 337-364.
- Diebold, F. X., Rudebusch, G. D., & Aruoba, B. S. (2007). The Macroeconomy and The Yield Curve: A Dynamic Latent Factor Approach. *Journal of Econometrics*, 131(1-2), 309-338.
- Dovis, A. (2019). Efficient Sovereign Default. *The Review of Economic Studies*, 86(1), 282-312.
- Fisher, M., Nychka, D., & Zervos, D. (1995). Fitting the Term Structure of Interest Rates With Smoothing Splines. *Finance and Economics Discussion Series, Federal Reserve Board, Working Paper 95-1*.
- Gallmeyer, M., Hollifield, B., Palomino, F. J., & Zin, S. E. (2007). Arbitrage-Free Bond Pricing with Dynamic Macroeconomic Models. *Federal Reserve Bank of St Louis*.
- IDX. (2019). *Indonesia Bond Market Directory 2018-2019*. Jakarta: Indonesia Stock Exchange - Indonesia Bond Pricing Agency.
- Imakubo, K., Kojima, H., & Nakajima, J. (2015). *The Natural Yield Curve: Its Concept and Measurement*. Tokyo: Bank of Japan Working Paper Series.
- Kemenkeu, D. (2019). *Direktorat Jenderal Pengelolaan Pembiayaan Dan Resiko Kementerian Keuangan*. Retrieved June 6, 2020, from <https://www.djppr.kemenkeu.go.id/#/id/page/debtstructure>
- Krishnamurthy, A., & Jorgensen, A. V. (2012). The Aggregate Demand for Treasury Debt. *Journal of Political Economy*, 120(2), 233-267.
- Li, W., & Song, Y. (2019). *The Term Structure of Liquidity Premium*. Los Angeles: USC Marshall School of Business Research Paper.
- McCulloch, H. J. (1975). The Tax-Adjusted Yield Curve. *The Journal of Finance*, 30, 811-830.
- Nelson, C. R., & Siegel, A. F. (1987). Parsimonious Modeling of the Yield Curve. *Journal of Business*, 6-(4), 473-489.
- Prathama, R., Sugiarto, Ugut, G., & Hulu, E. (2020). Pricing Model for Indonesia Government Bond. *Journal of Accounting*.
- Ullah, W., Matsuda, Y., & Tsukuda, Y. (2015). Generalized Nelson-Siegel term structure model: do the second slope and curvature factors improve the in-sample fit and out-of-sample forecasts? *Journal of Applied Statistics*, 42(4), 876-904.